Investment, speculation, and financial regulation

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Abstract

I study a production economy with financial markets ridden by speculation fueled by heterogeneous beliefs. Levels of disagreement far smaller than in forecast surveys lead to wealth volatility that cannot be matched in models with homogeneous beliefs. Wealth volatility depresses investment because pessimists often command a large wealth share and they invest cautiously. Through a series of numerical simulations I show that imposing leverage-like financial constraints on agents limits wealth movements, boosts investment, and significantly improves welfare.

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1 Introduction

The financial crisis of 2008 led to the failure of several large financial institutions which seemed to purposefully assume, rather than overlook, large risks. Some of the largest financial firms were liquidated, many others had to rely on government bail-out funds. How could key players in the financial markets make such risky investment decisions? Should there be more stringent restrictions on financial markets and the players in those markets? Would prior restrictions have prevented or at least tempered the crisis? Angelides [2011], reporting on behalf of the national financial crisis inquiry commission, concluded that "this crises was avoidable" and blamed "widespread failures in financial regulation." The senate committee investigating the financial crises also found regulatory failure to be one of its causes, see Levin and Coburn [2011].

The crisis also damaged the balance sheets of many households and individuals. Being heavily invested in equity and housing, they lost substantial amounts of wealth, and some now face difficulties financing their retirement years. At the same time, the IMF documents that investment declined steeply around the world, especially in the developed economies hit by the financial crisis.

I build upon a standard RBC model with complete financial markets and introduce heterogeneous beliefs like in Harrison and Kreps [1978]. Heterogeneous beliefs play an important role: they generate speculation. The latter is defined as financial market trade that is motivated by differences in opinion. For example, if one agent assigns higher probability to a recession than anyone else in the economy then his consumption share will grow if a recession does occur; otherwise his consumption share will shrink. In contrast, in an economy with homogeneous beliefs consumption share of each agent is constant. The reason why the agent's consumption share fluctuates when he disagrees with others is that his portfolio pays more than others' portfolios in a recession state and less in an expansion. That is the consumer makes a speculative bet on a recession and finances his trade by selling securities that pay in an expansion. Other consumers in the economy, disagreeing with him, are happy to accept such bet. The size of the financial bet depends on agents' risk-aversion and their wealth positions. But, however small, these trades never dissipate until only the agent with the most accurate beliefs remains in the market as pointed out by Sandroni [2000] and Blume and Easley [2006]. The so-called survival force provides the planner with reasons to guard agents from a financial ruin by imposing restrictions on the financial markets.

Belief heterogeneity would not matter if the financial markets were not sufficiently rich. For example, opportunities for speculation would be limited if financial markets only allowed investment in shares of physical capital. In this case the consumer that finds the investment more attractive would gradually build his ownership of capital by postponing his consumption. This would depress the return on capital and prompt other agents to bring their consumption forward as in Cogley et al. [forthcoming].

With heterogeneous beliefs and complete financial markets speculation thrives and individual consumption is extremely volatile. In fact, Blume and Easley [2009] show that individual consumption shares will infinitely often approach zero. With unbounded utilities the true, not subjective, welfare of consumers can be arbitrarily low. This raises the question if imposing financial constraints or limiting financial market access could increase the society's welfare. The answer would be no if the policy makers were guided by the Pareto criterion that uses subjective beliefs for in this case the competitive equilibrium allocation is optimal. Blume et al. [2014] argue that the welfare should be evaluated using the true probability distribution.[1] However, planner should not be granted knowledge of the true distribution. On the contrary, the planner just like market participants may have inaccurate beliefs. So, he must evaluate the society's welfare using the least favorable admissible true probability distribution. In this way both the planner and the consumers are subject to the same friction – a possibility of being endowed with incorrect beliefs.

The main result uses the welfare criterion introduced by Blume et al. [2014] and shows that as along as agents are sufficiently patient the society's welfare under the complete markets may be lower than under the financial autarky. Belief heterogeneity acts though two channels. First, it generates very volatile wealth distribution that triggers consumption volatility and lowers the society's welfare significantly. But an increase in wealth volatility also has indirect effect. For example, when the economy's productivity

[1] Gilboa et al. [2012] and Brunnermeier et al. [2012] propose different welfare criteria but they are not complete or transitive.

declines so does wealth of consumers that were betting on an expansion. Asset prices adjust "pessimistically" to reflect now increased wealth share of pessimists. Optimistic consumers cannot counteract this wave of pessimism as their wealth declines. As wealth concentrates in the hands of one group speculation slows down for the other group has little capacity to take the opposite side of bets. Hence, the economy suffers from more protracted recessions. This is the "persistence" channel. Then suppose that an economy experiences a boom and wealth is equally distributed. With heterogeneous beliefs the additional resources are more likely to be consumed then invested. This is so because consumers increase their speculative bets and all anticipating to profit in the future increase consumption already today. For this reason investment and asset prices also fluctuate more when wealth is distributed more evenly. This is the "speculation" channel.

As argued above belief heterogeneity affects the society's welfare adversely through several channels. But can this be remedied with financial restrictions? If so, what restrictions are more likely to be effective? To this end, I study a numerical example in which consumers are subjected to tight borrowing limits. The borrowing limits that we impose restrict the amount of debt that an agent could accumulate. Their purpose is to limit the amplitude of wealth fluctuation. This, in turn, constrains consumption variability. Mean investment increases marginally. It also becomes less volatile and less persistent. Thus, financial restriction can improve investment and welfare contrary to common wisdom. Although these gains come at the cost of more volatile asset prices suggesting that economic and financial stability goals may conflict.

In the next section I give a brief overview of related literature. In section 2 I describe the model setting and analyze several special cases shedding light on the effect of wealth inequality and volatility. Section 3 discusses why regulation proves beneficial and section 4 describes the quantitative example. I use concluding remarks to discuss limitations of the analysis and point to possible extensions.

1.1 Related work

The closest work to ours is Cao [2011] and Baker et al. [2013]. Both study production economies with heterogeneous beliefs but their motivation concerns asset pricing puzzles. We instead focus on the welfare implications of belief heterogeneity. Cao [2011] analyzes an economy with collateral constraints. But they play rather a technical role of insuring equilibrium existence. And unfortunately his proof of equilibrium existence does not extend to the present framework. Baker et al. [2013] approach cannot be adapted to analyze occasionally binding constraints like in our work. So, it is not suitable for an analysis of restricted financial market structures. Further, production technology in their model is linear and, hence, the competitive equilibrium resembles closely that of an endowment economy. Fostel and Geanakoplos [2008] show that an introduction of a new asset class may not benefit the issuer if it is being purchased by "anxious" investors who manage little wealth and who have significant risk exposure. Like in the work of Hart [1975] opening a new market may lower welfare if markets are incomplete to start with. That is in our work financial restrictions are desirable ex-ante as opposed to being a discretionary response to ex-post shock.

2 Model

Time and uncertainty. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. At each date a state is drawn from the set $\mathcal{S} = \{1, \ldots, S\}$. The set of all sequences of states is denoted by Σ with a representative sequence $\sigma = (s_0, s_1, ...)$, called a path. Let $\sigma^t = (s_0, ..., s_t)$ denote the partial history through date t. We use $\sigma'|\sigma^t$ to indicate that a path σ' coincides with a path σ up to and including period t.

The set Σ together with its product sigma-field defines the measurable space on which everything is built. Let P^0 denote the "true" probability measure on Σ. For any probability measure P on Σ, $P_t(\sigma)$ is the (marginal) probability of the partial history $\sigma^t : P_t(\sigma) = P(\{\sigma^t\} \times S \times S \times \cdots)$.

In the next few paragraphs we introduce a number of random variables of the form $x_t(\sigma)$. All such random variables are assumed to be date-t measurable; that is, their value depends only on the realization of states through date t. Formally, $\mathcal{F}_t$ is the σ-field of events measurable at date t, and each $x_t(\sigma)$ is assumed to be $\mathcal{F}_t$-measurable.

Firms and technology. A large number of competitive firms produces consumption goods according to a CRS technology $z_t(\sigma)F(K, L)$ where $z_t(\sigma)$ denotes the aggregate productivity, K and L denote respectively the aggregate quantity of capital and labor hired. A firm pays ongoing competitive rates for capital and labor, respectively $r_t(\sigma)$ and $w_t(\sigma)$. Let $Y_t(\sigma)$ denote the aggregate production of consumption goods in period t along path σ.

Capital goods are produced according to a linear technology using consumption goods only. It takes one unit of consumption good to produce one unit of a capital good and vice versa. Used capital depreciates at rate $\delta \in (0, 1]$.

We assume the following properties of the aggregate productivity:

A1. *The aggregate productivity is uniformly bounded from above and away from* 0*:*

$$\infty > \bar{z} = \sup_{t,\sigma} z_t(\sigma) \geqslant \inf_{t,\sigma} z_t(\sigma) = \underline{z} > 0.$$

Financial markets trade S Arrow securities. An Arrow security j purchased in period t pays one unit of consumption in period $t + 1$ if state $\sigma_{t+1} = j$ realizes and it is valued at $Q_t(j|\sigma)$. Trading is subject to the natural borrowing limits defined later. Consumer i's period t investment in security j is denoted by $a_t(\sigma)$.

Consumers, beliefs, preferences. An economy contains I consumers, each with consumption set $\mathbb{R}_+$. A *consumption plan* $c : \Sigma \to \prod_{t=0}^{\infty} \mathbb{R}_+$ is a sequence of $\mathbb{R}_+$-valued functions $\{c_t(\sigma)\}_{t=0}^{\infty}$ in which each $c_t(\sigma)$ is $\mathcal{F}_t$-measurable.

Consumer i is endowed with $l_t^i(\sigma) > 0$ units of labor in each period t along every path $\sigma \in \Sigma$ that supplied inelastically to the labor market.

A *consumption plan* $c : \Sigma \to \prod_{t=0}^{\infty} \mathbb{R}_+$ is a sequence of $\mathbb{R}_+$-valued functions $\{c_t(\sigma)\}_{t=0}^{\infty}$ in which each $c_t(\sigma)$ is $\mathcal{F}_t$-measurable.

An *investment plan* $x : \Sigma \to \prod_{t=0}^{\infty} \mathbb{R}$ is a sequence of $\mathbb{R}$-valued functions $\{x_t(\sigma)\}_{t=0}^{\infty}$ in which each $x_t(\sigma)$ is $\mathcal{F}_t$-measurable. Consumer i is endowed in period 0 with a positive amount of capital k_0^i.

A *trading plan* $a : \Sigma \to \prod_{t=0}^{\infty} \mathbb{R}^S$ is a sequence of $\mathbb{R}^S$-valued functions $\{a_t(\sigma)\}_{t=0}^{\infty}$ in which each $a_t(\sigma)$ is $\mathcal{F}_t$-measurable. Each consumer starts with a zero position in each of the Arrow securities.

An *allocation* is a profile of consumption, investment and trading plans for each individual. The allocation $((c^1, x^1, a^1), \ldots, (c^I, x^I, a^I))$ is *feasible* if for all σ and t, $\sum_i [c_t^i(\sigma) + x_t^i(\sigma)] - Y_t(\sigma) = 0$ and $\sum_i a_t^i(\sigma) = 0$.

Consumer i's preferences on consumption plans are described by a *belief* P^i, a probability distribution on Σ, a discount factor $0 < \beta < 1$, and a payoff function $u : \mathbb{R}_{++} \to \mathbb{R}$. Leisure is not valued. The utility that consumer i assigns to consumption plan c is the expectation of the discounted value of the sequence of payoff realizations:

$$U_{P^i}(c) = (1-\beta) E_{P^i} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t(\sigma)) \right\}. \tag{1}$$

Beliefs are consumer-specific. Consumers' beliefs need not coincide with what will actually happen. The *true* probability distribution is denoted by P^0. We will impose some constraints on how different beliefs can be. The proposed setup allows us to analyze a wide range of belief specifications, including beliefs that evolve according to some learning rule. But for expositional purposes we restrict our attention to beliefs that are generated by time-homogeneous Markov chains. Let P^i be constructed from a Markov transition matrix $\Pi^i = \{\pi^i_{jk}\}$. Then:

$$P^i_t(\sigma|\sigma_0) = \prod_{\tau=1}^{t} \pi^i(s_\tau|s_{\tau-1}). \tag{2}$$

We call so-constructed belief system *dogmatic* because consumer i never updates his forecast distribution. We also prohibit learning from prices and other endogenous variables. The structure of the environment, including the beliefs, is a common knowledge as in Radner [1968].

We measure distance between beliefs using relative entropy, or Kullback-Liebler divergence. The relative entropy of belief P^i is:

$$\mathcal{KL}(P^i|P^0) = \sum_j \bar{\pi}^0_j \sum_k ln(\pi^0_{jk}/\pi^i_{jk}), \tag{3}$$

where $\bar{\pi}^0 = \{\bar{\pi}^0_j\}$ is the stationary distribution of a Markov chain with transition matrix Π^0. It is easy to verify that $\mathcal{KL}(P^i|P^0) \geqslant \mathcal{KL}(P^0|P^0) = 0$.

Definition. Beliefs of a type-i agent are more accurate than beliefs of a type-j agent if

$$\mathcal{KL}(P^i|P^0) \leqslant \mathcal{KL}(P^j|P^0).$$

We assume the following properties of the payoff function:

A2. *Each* $u : \mathbb{R}_{++} \to (-\infty, \infty)$ *is* C^1*, strictly increasing, strictly concave, and satisfies an Inada condition at 0:* $\lim_{c \downarrow 0} u_i'(c) = \infty$.

We assume the following properties of the aggregate labor supply:

A3. *The aggregate labor supply is uniformly bounded from above and away from* 0*:*

$$\infty > \bar{L} = \sup_{t,\sigma} L_t(\sigma) \geqslant \inf_{t,\sigma} L_t(\sigma) = \underline{L} > 0.$$

2.1 Competitive equilibrium

A *competitive equilibrium* is a feasible allocation and a price system such that ...

Firm's optimization problem. Because the technology is CRS we study a representative firm. The firm's profit maximizing choice implies the following input prices:

$$r_t(\sigma) = z_t(\sigma) F_k(K_{t-1}(\sigma), L_t(\sigma)) + 1 - \delta,$$
$$w_t(\sigma) = z_t(\sigma) F_l(K_{t-1}(\sigma), L_t(\sigma)).$$

The aggregate output of the economy is:

$$Y_t(\sigma) = z_t(\sigma) F(K_{t-1}(\sigma), L_t(\sigma)). \tag{4}$$

Aggregate capital and labor supply. The aggregate labor supply evolves exogenously and the aggregate capital evolution is given by:

$$K_t(\sigma) = (1 - \delta) K_{t-1}(\sigma) + X_t(\sigma). \tag{5}$$

Market clearing. The goods market clearing condition is:

$$\sum_i [c_t^i(\sigma) + x_t^i(\sigma)] = z_t(\sigma) F(K_{t-1}(\sigma), L_t(\sigma)) + (1 - \delta) K_t(\sigma).$$

The capital and labor market clearing conditions are:

$$L_t(\sigma) = \sum_i l_t^i(\sigma),$$
$$K_t(\sigma) = \sum_i k_t^i(\sigma).$$

The financial market clearing condition is:

$$0 = \sum_i a^i_{t+1}(\sigma').$$

Consumer's optimization problem. Consumer i's budget constraint is:

$$c^i_t(\sigma) + k^i_t(\sigma) + \sum_{\sigma'|\sigma^t} Q_t(\sigma')a^i_{t+1}(\sigma') = a^i_t(\sigma) + \underbrace{r_t(\sigma)k^i_{t-1}(\sigma) + w_t(\sigma)l^i_t(\sigma)}_{\textit{capital and labor income}} \quad (6)$$

Purchases of Arrow securities are subject to the natural borrowing limits:[2]

$$a^i_{t+1}(\sigma') \geqslant -N^i_{t+1}(\sigma'), \quad \forall \sigma'|\sigma^t, \quad (7)$$

Natural borrowing limits never bind in a competitive equilibrium since the period utility function satisfies the Inada condition stated in assumption A1. Consumer i chooses consumption, investment and asset trading plans to maximize life-time utility (1) subject to constraints (6) and (7).

The first-order necessary optimality conditions are:

$$Q_t(\sigma') = \beta \pi^i_t(\sigma'|\sigma)\left[\frac{c^i_{t+1}(\sigma')}{c^i_t(\sigma')}\right]^{-\gamma}, \quad (8a)$$

$$1 = \beta \sum_{\sigma'} \pi^i_t(\sigma'|\sigma)\frac{u'(c^i_{t+1}(\sigma'))}{u'(c^i_t(\sigma'))} r_{t+1}(\sigma'). \quad (8b)$$

Let the consumption share of agent i be denoted by $s^i_t(\sigma)$:

$$s^i_t(\sigma) \equiv \frac{c^i_t(\sigma)}{\sum_j c^j_t(\sigma)}. \quad (9)$$

Then recognizing that all agents are facing the same prices we get:

$$s^i_{t+1}(\sigma') = \frac{[p^i_t(\sigma'|\sigma^t)]^{1/\gamma} s^i_t(\sigma)}{\sum_j [p^j_t(\sigma'|\sigma^t)]^{1/\gamma} s^j_t(\sigma)}, \quad \forall \sigma'|\sigma^t. \quad (10)$$

[2] Define the j-period ahead price $Q^j_t(\sigma) = \Pi^{j-1}_{k=0} Q_{t+k}(\sigma)$. Then a natural borrowing limit equals the date-t value of the continuation of a consumer's labor income plan:

$$N^i_t(\sigma) = \sum_{j=0}^{\infty} \sum_{\tilde{\sigma}|\sigma^t} Q^j_t(\tilde{\sigma}) w_{t+j}(\tilde{\sigma}) l^i_{t+j}(\tilde{\sigma}).$$

Then the Arrow security price can be expressed in the following way:

$$Q_t(\sigma') = \beta \cdot \underbrace{\Big[\sum_j [p_t^j(\sigma'|\sigma^t)]^{1/\gamma} s_t^j(\sigma)\Big]^\gamma}_{\textit{CES belief aggregator}} \cdot \Big[\frac{C_{t+1}(\sigma')}{C_t(\sigma)}\Big]^{-\gamma}. \tag{11}$$

In what follows we refer to the middle term interchangeably as population belief or belief aggregate:

$$\widetilde{p}_t(\sigma'|\sigma^t) \equiv \Big[\sum_j [p_t^j(\sigma'|\sigma^t)]^{1/\gamma} s_t^j(\sigma)\Big]^\gamma.$$

The Arrow security price contains familiar terms: discount factor, "population belief", and growth rate of the marginal utility. The population belief is a CES (constant elasticity of substitution) aggregate of individual beliefs with the weights being the individual consumption shares. While it is tempting to refer to the population beliefs as "probabilities" they do not sum to one implying interesting new effects that are explained next.

2.2 Belief vs wealth dispersion

Consider the case with $\gamma \geqslant 1$. If the average belief in the population is correct, $(1/I)\sum_j p_t^j(\sigma'|\sigma^t) = p_t^0(\sigma'|\sigma^t)$, then:

$$\Big[\sum_j [p_t^j(\sigma'|\sigma^t)]^{1/\gamma} s_t^j(\sigma)\Big]^\gamma \leqslant p_t^0(\sigma'|\sigma^t).$$

The above inequality holds strictly as long as at least one of the probabilities is different from the truth. That is the aggregated probabilities underestimate the true probabilities for every state. The consequence of the above is equivalent to that of an increase in impatience, that is lower β. In turn, investment in capital, or any asset in positive net supply, is also lower.

One could measure the degree of belief dispersion/heterogeneity by:

$$\textit{belief dispersion} \equiv \sum_j [p_t^j(\sigma'|\sigma^t) - p_t^0(\sigma'|\sigma^t)]^2 s_t^j(\sigma). \tag{12}$$

Lemma 1 implies that the discrepancy between the truth and the belief aggregate increases as the dispersion of beliefs increases.

Lemma 1. *If the two belief assignments p and q are such that $\sum_j p^j_t(\sigma'|\sigma^t)s^j_t(\sigma) = \sum_j q^j_t(\sigma'|\sigma^t)s^j_t(\sigma)$ and $\sum_j[p^j_t(\sigma'|\sigma^t)]^2 s^j_t(\sigma) > \sum_j[q^j_t(\sigma'|\sigma^t)]^2 s^j_t(\sigma)$ (p is more dispersed) then:*[3]

$$\widetilde{p}(\sigma'|\sigma^t) \begin{array}{l} < \widetilde{q}(\sigma'|\sigma^t) \\ = \widetilde{q}(\sigma'|\sigma^t) \\ > \widetilde{q}(\sigma'|\sigma^t) \end{array} \quad \text{if} \quad \begin{array}{l} \gamma > 1 \\ \gamma = 1 \\ \gamma < 1 \end{array} . \tag{13}$$

We will also need another related result. Lemma 2 states that in the case with two consumers dispersion of consumption shares has a similar effect to that of belief dispersion.

Lemma 2. *If the consumption shares of consumers are $(x, 1-x)$ then*

$$g(x) \equiv \sum_{\sigma'|\sigma^t} \widetilde{p}(\sigma'|\sigma^t) \equiv \sum_{\sigma'|\sigma^t} [x(p^1(\sigma'|\sigma^t))^{1/\gamma} + (1-x)(p^2(\sigma'|\sigma^t))^{1/\gamma}]^\gamma$$

is convex/constant/concave function if γ is larger/equal/smaller than 1. Moreover, for all $x \in (0,1)$ we have:

$$\sum_{\sigma'|\sigma^t} \widetilde{p}(\sigma'|\sigma^t) \begin{array}{l} < 1 \\ = 1 \\ > 1 \end{array} \quad \text{if} \quad \begin{array}{l} \gamma > 1 \\ \gamma = 1 \\ \gamma < 1 \end{array} . \tag{14}$$

Proof. Without loss of generality assume that $\gamma > 1$. Then straightforward differentiation shows that $g'(0) < 0, g'(1) > 0$. Another round of differentiation shows that $g''(x) > 0, \forall x$. Using the fact that $g(0) = g(1) = 1$ one obtains $g(x) < 1, \forall x \in (0,1)$. □

The implications are obvious if we keep in mind that $\sum_{\sigma'|\sigma^t} p^0(\sigma'|\sigma^t) = 1$. If $\gamma > 1$ then the belief aggregate is pessimistic in the sense that it assigns a lower "expected" value to any positive payoff. The result about the shape implies that for $\gamma > 1$ an increase in consumption share dispersion brings the economy closer to the homogeneous beliefs benchmark as term $\sum_{\sigma'|\sigma^t} \widetilde{p}(\sigma'|\sigma^t)$ approaches 1. For this reason wealth dynamics, that drives the consumption inequality, is an important determinant of investment.

[3] The proof of this claim follows directly from Jensen's inequality and is thus left without a proof.

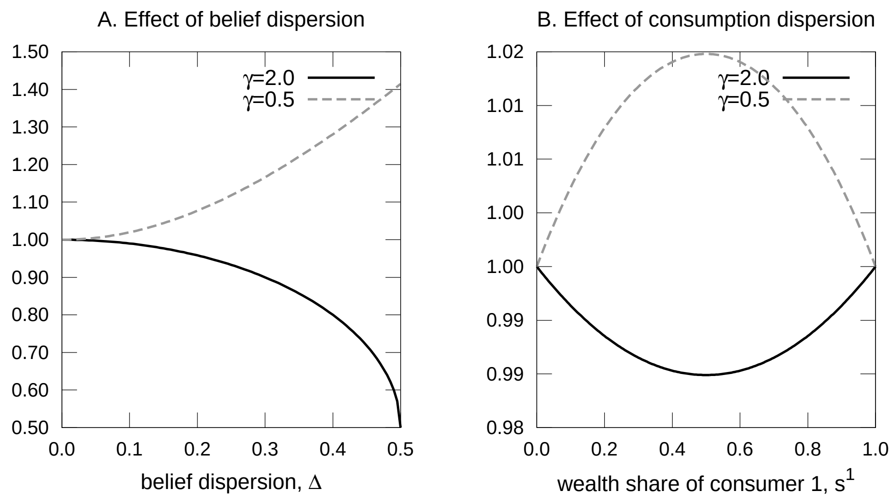

Figure 1: Belief aggregate $[s^1(0.5-\Delta)^{1/\gamma}+(1-s^1)(0.5+\Delta)^{1/\gamma}]^\gamma$. Panel A fixes $s^1 = 0.5$ and aries Δ. Panel B fixes $\Delta = 0.1$ and varies s^1.

2.3 Analytical results for a two-period model

The main question is "How does the belief dispersion affect investment?" We demonstrate two channels, direct and indirect, that are at work using a two-period version of the model. In what follows we study the case with $\gamma > 1$, the assumption that is motivated later. At the end of this section we discuss how the results depend on this assumption. We also assume that the belief assignment is "symmetric." It is easiest to explain this concept for the case with two consumers. In this case beliefs are said to be symmetric if for any state j there exists state k such that the aggregate productivity and the employment profile are the same in the two states but $p^1(j|\sigma^t) = p^2(k|\sigma^t)$. To illustrate, suppose that there are two states of the world and that the aggregate productivity and employment profiles are constant across these states. Then the belief assignment is symmetric if the probability that consumer 1 assigns to state 1 equals the probability that consumer 2 assigns to state 2.

Proposition 1 (Belief dispersion effect)**.** *If $\gamma > 1$ in the symmetric two-period model an increase in belief dispersion lowers aggregate investment.*

Proof. The Euler equation for capital accumulation is:

$$1 = \beta \sum_s \widetilde{p}_s \left[\frac{C_{1s}}{C_0}\right]^{-\gamma} [z_{1s} F_k(K_0, L_{1s}) + 1 - \delta].$$

Using the good market clearing condition we can substitute out the aggregate consumption in the above: $C_0 = z_0 F(K_{-1}, L_0) + (1-\delta)K_{-1} - K_0, C_{1s} = z_{1s} F(K_0, L_{1s}) + (1-\delta)K_0$. It is an implicit equation for K_0, but it depends on the equilibrium distribution of consumption shares. Assuming that consumers are *ex-ante* identical, which means that their beliefs are symmetric as explained above, then an increase in belief dispersion does not affect individual consumption shares. Each consumer's share must equal $1/I$. Then by lemma 1 if the dispersion of beliefs were increased the right-hand side of the Euler equation would decrease. So, K_0 must decrease to restore equality. □

Several remarks are in order. First, the symmetry of the belief assignment is needed to insure that consumption in period 0 does not respond to an increase in belief dispersion. But it is reasonable to expect that this additional effect would not be dominant. Second, if learning from past observations of the exogenous state were allowed this effect would vanish only asymptotically, because a true process generally is learned only asymptotically. Third, observe the role that the preference homotheticity plays. In the above analysis only dynamics of the aggregate macroeconomic indicators matters.

The above argument holds for $\gamma > 1$. If $\gamma = 1$ and the population average belief is correct then belief heterogeneity has no effect on the equilibrium investment. If $\gamma < 1$ then there will be over-investment that is also inefficient.

To demonstrate the second effect we need to introduce new concessions. We assume that there is full depreciation: $\delta = 1$. This simplifies substantially the Euler equation for capital. The additional element that is needed is heterogeneity of the initial capital stock ownership. For clarity assume that there are only two consumers. If we redistribute the initial capital from consumer 1 to consumer 2 then consumption share of the former must decrease. Let the consumption share of consumer 1 be $0.5 - \Delta$. We are interested in the impact of an increase in Δ on the aggregate investment.

Wealth dispersion is more difficult to analyze as it impacts the belief aggregate differently in different states. In contrast, an increase in the belief dispersion affected the belief aggregate in the same direction. This prompts an additional assumption – no aggregate uncertainty. In this case aggregate investment depends only on $\sum_s \widetilde{p}_s$ and lemma 2 can help with this object.

Proposition 2 (Wealth dispersion effect). *If $\gamma > 1$ in the symmetric two-period model an increase in consumption dispersion increases aggregate investment.*

Proof. Under the assumptions of the proposition the Euler equation for capital accumulation is:

$$1 = \beta \sum_s \widetilde{p}_s \Big[\frac{C_{1s}}{C_0}\Big]^{-\gamma} F_k(K_0, 1) = \frac{\beta \alpha_k [F(K_0, 1)]^{1-\gamma}}{K_0[z_0 F(K_{-1}, 1) - K_0]^{-\gamma}} \sum_s \widetilde{p}_s.$$

where α_k is the share of capital in the production cost. The last equality makes use of the good market clearing condition. The effect on K_0 depends on $\sum_s \widetilde{p}_s$. Lemma 2 states that for $\gamma > 1$ we have $\sum_s \widetilde{p}_s < 1$ and that wealth dispersion brings this object closer to 1. Because the right hand side is a decreasing function of K_0 wealth dispersion has a positive impact on investment. □

This result is surprising. One may conclude that it is beneficial to let speculation in. This is not so and the reason why investment and wealth dispersion are positively correlated is important. When wealth is more dispersed speculation opportunities are limited. There are two reasons. First, asset prices move closer to the valuation of wealthy investors and the latter's incentives to speculate decrease. Second, poor investors speculative demand decreases, despite their incentives to speculate increasing, because they have little wealth. As a consequence, investors purchase less risky portfolios of Arrow securities allowing them to make a larger investment in physical capital.

This result depends on the fact that there is no aggregate uncertainty. With aggregate uncertainty there is an additional force: the effect of wealthy optimists on aggregate investment is smaller than that of wealthy pessimists. We will analyze the case with aggregate risk using a numeric example in section 4.

2.4 Risk-aversion vs IES

The evolution of the individual consumption shares depends on what we referred to as risk-adjusted probabilities $[p_t^i]^{1/\gamma}$. But the parameter γ has two interpretations under this preference model: a risk-aversion coefficient or the inverse of the IES. To determine which of the two roles γ plays we employ the two-period model again. This time, however, we impose Epstein and Zin [1989] preference specification that allows to disentangle the risk and temporal aspects of individual behavior. Consumer i ranks different consumption plans according to:

$$\left[c_0^\rho + \beta\Big(E^i[c_{1s}^\alpha]\Big)^{\frac{\rho}{\alpha}}\right]^{\frac{1}{\rho}},$$

where s indexes possible states. The risk-aversion coefficient for this preference specification is $\frac{1}{\alpha-1}$ and the IES is $\rho - 1$.

The first-order conditions for consumer i yield the following expression for the price of Arrow security s:

$$Q_s = \beta p_s^i \left[\frac{c_{1s}^i}{c_0^i}\right]^{\rho-1} \left[\frac{(c_{1s}^i)^\alpha}{E^i[(c_{1s}^i)^\alpha]}\right]^{\frac{\alpha-\rho}{\alpha}}. \tag{15}$$

If α and ρ were equal then the last term in the above expression would disappear yielding equation (8a) as expected. Next assume that the setting is symmetric, that is $c_0^i = c_0^j, E^i[(c_{1s}^i)^\alpha] = E^j[(c_{1s}^j)^\alpha]$, $\forall i, j$. Then recognizing that all consumers face the same price we get:

$$s_{1s}^i = \frac{[p_s^i]^{\frac{1}{\alpha-1}} s_0^i}{\sum_j [p_t^j]^{\frac{1}{\alpha-1}} s_0^j} \tag{16}$$

The transformation of individual beliefs involves the risk-aversion coefficient alone. That is they were rightfully called risk-adjusted probabilities in the previous sections. So, the effect of belief dispersion effect that is described in proposition 1 depends only on the risk-aversion parameter. This proves proposition 3.

Proposition 3. *In the symmetric two-period model with Epstein-Zin preferences the belief aggregate depends on the risk-aversion but not on the IES coefficient.*

The above analysis then raises a question about the role of the IES? Intuitively, (inverse of) the IES governs the strength of the asset price response to changes in the inter-period relative consumption. With a lower IES asset prices must be more volatile. Because prices are more volatile so must be individual financial wealth. Hence, we should see a stronger wealth dynamics effect described in proposition 2.

3 Need for regulation

We now assess the effect on welfare. We start the analysis from the evolution of the consumption shares. From equation (9) it follows that:

$$\exp(\eta_t(\sigma)) \equiv \frac{s_t^i(\sigma)}{s_t^j(\sigma)} = \left[\frac{p_t^i(\sigma^t)}{p_t^j(\sigma^t)}\right]^{1/\gamma}, \quad \forall i, j.$$

Then:

$$\frac{1}{t}\eta_t(\sigma) = \gamma\frac{1}{t}\left[\log(p_t^0(\sigma)/p_t^j(\sigma)) - \log(p_t^0(\sigma)/p_t^i(\sigma))\right]. \qquad (17)$$

When agents believe that the state process is a finite Markov chain then the right hand side converges p^0 almost surely to the difference of relative entropies $\mathcal{KL}(p^j|p^0) - \mathcal{KL}(p^i|p^0)$. If the latter is positive[4] then, Blume and Easley [2009] show, η_t must diverge which is possible only if consumer j consumption share converges to zero, that is he is driven out of the market. Mathematically, $\lim \sum c_t^i(\sigma) = 0$ almost surely with respect to the true probability distribution p^0.

Unlike in Blume and Easley's analysis the aggregate output is endogenous in this work. So far, it could only be established that consumption *share* of the agent with less accurate beliefs converges to zero p^0 almost surely. But it is easy to establish an upper bound on the aggregate capital stock. The highest level of investment would be achieved in the case when the most optimistic consumer amasses all the wealth in equilibrium. The affect of disagreement disappears as all other consumers have zero impact on the equilibrium prices. The upper bound $\bar{K}$ then solves:

$$1 = \beta[\max_{t,\sigma} z_t(\sigma)F'(\bar{K}, \max_{t,\sigma} L_t(\sigma)) + 1 - \delta].$$

[4] That is consumer i has more accurate beliefs.

The aggregate output is bounded above and if a consumer's consumption share converges to zero then his consumption level must also converge to zero.

The more interesting situation is when consumers have equally accurate beliefs. In that case each consumer's consumption share will be p^0 infinitely often arbitrarily close to zero. Importantly, the expected time until the event $s_t^i(\sigma) < \epsilon$ depends only on the exogenous relative likelihood process. Assuming that $\gamma > 1$ this implies that his utility can be arbitrarily low. In view of the above restricting consumers' access to financial markets may prove beneficial for if noone could trade in the Arrow-security markets welfare level in the economy would be finite.

[AN ARGUMENT FROM SANTOS 2000 CAN BE USED TO ESTABLISH A LOWER BOUND ON K.]

3.1 Welfare criterion

Blume et al. [2014] argue that in the environment with heterogeneous beliefs usage of the Pareto criterion is not reasonable. The reason is that the same allocation is evaluated differently by each individual. That is even if agents have the same preferences but different beliefs a change of allocation from x to x' may be perceived as welfare improving by one consumer but not the other. That is many, in fact most of, allocations cannot be ranked consistently by both agents.

It is also possible that when both consumers agree to a move from x to x' even though it must definitely harm one of them because both cannot be correct. This issue, referred to as spurious anonymity, has been described by Mongin [2005]. In the context of the current model all consumers prefer the complete markets to any other financial arrangement. This is so because each of them expects to profit at the expense of the other. Yet, this is not possible and someone's welfare must be hurt. And, as it will be shown using numerical examples in section 4, welfare costs can be huge.

The criterion proposed by Blume et al. [2014] avoids these problems by dispensing with the individual welfare evaluations. Instead the authors advocate to use the true distribution to compute individual welfare levels, which may be disagreeable for consumers. However, this raises a question of why wouldn't the paternalistic planner inform consumers of what the true process is. The authors assume that the planner is not confident that he has superior information. The motivation for this is that with finite macroeconomic series it is difficult to distinguish a random walk from a highly persistent auto-regressive process, or processes with long-run risk as in Bansal and Yaron [2004] or disasters as in Rietz [1988] and Barro [2006]. This motivates him testing many plausible assignments of the truth and beliefs across consumer types. The welfare is then computed using the least favorable assignment of belief and the true distribution:

$$\min_{p^0,p^1,p^2,\ldots,p^I} \sum_i U_{p^0}(c^i(p^1,p^2,...,p^I)). \tag{18}$$

It is made explicit that the consumption plans depend on the belief assignment $(p^1, p^2, ..., p^I)$. However, life-time utilities are computed using the least favorable true distribution.

Proposition 4. *Consider an infinite horizon economy in which the productivity and the individual labor supplies are fixed at 1 and the initial capital stock ownership of each consumer is $\bar{K} \equiv ((\rho+\delta)/\alpha)^{\frac{1}{1-\alpha}}$. Suppose that beliefs of consumer 1 are the least accurate. Then there exists $\bar{\beta}$ such that if $\beta > \bar{\beta}$ then the true society welfare under complete markets is lower than under the financial autarky.*

Proof. First, consider the financial autarky. Because consumers cannot disagree on investment profitability they all invest the same amount: $x_t^i(\sigma) = \delta\bar{K}$. Consumption of each consumer is constant: $c_t^i(\sigma) = \bar{C} \equiv \bar{K}^\alpha - \delta\bar{K}$. Life-time utility, true or subjective, of each consumer is $u(\bar{C})$.

Second, consider the complete markets. The analysis in Blume and Easley [2006] implies that $\limsup_t s_t^1(\sigma) = 0$. Observe that the evolution of the consumption shares depends on the properties of the relative likelihood ratio only. Then for any small $e > 0$ there exists T_e such that $s_t^i < e$ for all $t \geqslant T_e$. This implies that the life-time utility can be bounded above by:

$$(1-\beta^{T_e})u(Y_{max}) + \beta^{T_e}u(eY_{max}).$$

Since T_e is independent of β the above lower bound can be made arbitrarily small by increasing β. This proves the claim. □

The above proposition states that if consumers are sufficiently patient then for any assignment of beliefs the society's welfare will be lower under the complete financial markets than under the financial autarky. This suggests that partially restricting the financial markets may yield welfare that is higher in than under the either of the two extreme financial market structures. We study the magnitude of possible gains using a numerical example.

4 Quantitative example

We study the following example. There are only two states of the world. The true data generating process is iid that assigns equal probability to each state. Consumers 1 and 2 believe the probability distribution to be $[0.45, 0.55]$ and $[0.55, 0.45]$ respectively. Productivity levels in the two states are $\{z_l, z_h\} = \{0.97, 1.03\}$. Labor supply of each consumer is 1 irrespectively of the state. We assume that $\gamma = 2, \beta = 0.96$, both commonly-assumed values. Because there are only two states it is enough to introduce two assets to complete the financial markets. We choose the risk-free bond to be traded in addition to equity shares. Notice that it must be allowed for consumers to take negative positions in equity as otherwise markets would not be complete.

The state of the economy can be summarized by the triplet (w^1, K, z) where K, z are the aggregate capital stock and productivity level. Wealth share ω^1 is the wealth share of consumer 1 that is defined as follows:

$$\omega_t^1(\sigma) \equiv \frac{R_t(\sigma)k_t^1(\sigma) + W_t(\sigma) + b_t^1(\sigma)}{Y_t(\sigma) + (1-\delta)K_t(\sigma)}. \tag{19}$$

Policy functions are reported in figure 2. Current aggregate capital stock and productivity state are assumed to be $(\bar{K}, z_h)$ where $\bar{K}$ is the steady state level of capital stock in the economy with homogeneous beliefs. In the homogeneous beliefs benchmark the solution does not depend on the wealth distribution. Panel A then presents the solution as percentage deviations relative to the benchmark. Aggregate investment is depressed resulting in the lower capital stock. As the wealth share of consumer 1 approaches 0.5 (even distribution) investment decreases. This is so because as the wealth distribution becomes more evenly distributed speculation motives strengthen. Anticipating future financial results consumers increase consumption above the benchmark level. Observe also that the effects are not very large – capital

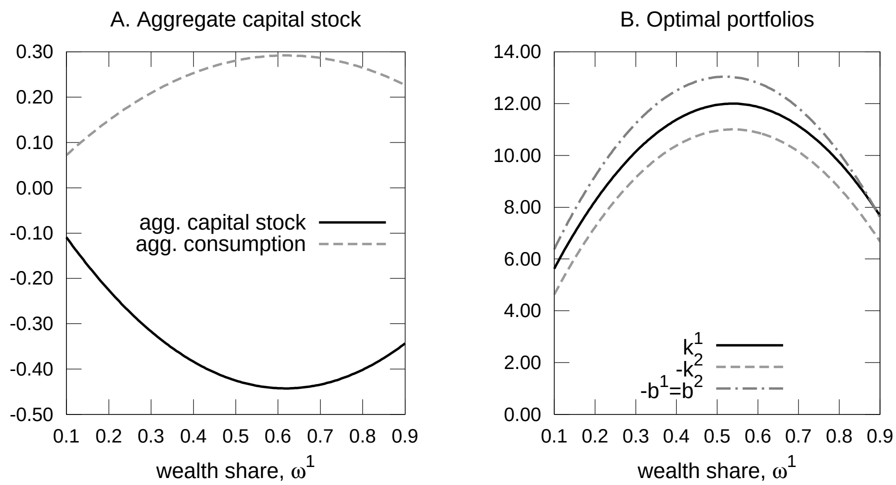

Figure 2: Aggregate capital stock, consumption and the optimal portfolios. Panel A reports percentage deviations from homogeneous beliefs benchmark. Panel B reports the optimal portfolios. Current aggregate capital stock and productivity state are assumed to be $(\bar{K}, z_h)$.

stock is depressed by only 0.45% which means that the aggregate production loses only 0.15%. The lowest level of investment is however reached at $\omega^1 \approx 0.62$ because of the productivity differences across the two states. At $\omega^1 = 0.5$ the pessimistic consumer 2 has a stronger effect on the asset prices because marginal utilities are larger in the low productivity state. Panel B plots the optimal portfolios. The optimistic consumer 1 invests approximately 4 times the aggregate capital stock in equity and shorts the risk-free bond. The pessimistic consumer 2 does the opposite. The difference between the k^1 and $-k^2$ lines is the aggregate capital stock that is used to produce goods. Notice the extreme positions taken by the consumers. They are such because there is little disagreement about the value of capital and the bond and so large positions are needed to take advantage of belief differences. The positions would be smaller if the consumers were allowed to trade Arrow securities instead.

A note of caution must be given. The above description seems to suggest that consumption actually increases. However, one should take also the dynamic reaction into the account: investment is depressed and output next period must be lower. In fact, the mean[5] capital stock is 0.52% lower in the heterogeneous beliefs economy. The average consumption is also lower but by a mere 0.15%. That is consumption share in the economy's GDP increases by 0.37%. The 0.15% loss of the mean consumption translates into 0.15% loss of welfare. But the major loss of utility comes from increases consumption volatility.[6] The overall welfare impact is equivalent to a 1.44% permanent loss of consumption.

4.1 Wealth dynamics as a propagation channel

To understand further the effect of belief heterogeneity consider the following "crisis" example. For this example we modify the state process. State 1 (expansion or "E") is assumed to occur with probability 0.90 and state 2 (contraction or "C") with probability 0.10. Beliefs over the two states are (0.95,0.05) and (0.85,0.15) for type-1 and type-2 consumers respectively. Under this new state process the Arrow security paying in the contraction state is cheap, as its price is proportional to the aggregate belief about this state. But there is also a second, more important, effect: relative disagreement about the contraction state is stronger. The relative disagreement about the contraction state is $|p^1(C) - p^2(C)|/p^0(C) = 1.00$ while it is only 0.11 for the expansion state. Hence, trades are larger in this market.[7] Consider now the following path of productivity: $E, E, E, E, E, C, C, E, ...$ Figure 3 plots the consumption share and the aggregate investment impulse responses. The initial wealth share of type-1 consumers is chosen so that this group's consumption share is 0.5. The evolution of the consumption share could be computed without solving the model as it is driven solely by the dynamics of the relative likelihood that the two groups attach to the assumed path of events. The type-1 consumer that attaches higher

[5] We simulate 10,000 series of length 100 and compute the cross-sectional average for the last period.

[6] There is no stationary distribution – the economy eventually spends all the time at the two extremes with one of the agents commanding zero wealth. Period 100 volatility of consumption increases 88.3%.

[7] A good analogy is trade in out-of-money options. They cost fractions of a penny and are heavily traded.

probability to the expansion state gains wealth and sees his consumption share increase each time the expansion state realizes. But, as pointed out above, the trades in the expansion state security are relatively small for the disagreement is relatively small. So, little wealth is transferred between consumers in expansions. The opposite is true in contractions. For this reason consumption share of type-1 consumers declines nearly 30 per cent in just two periods. Despite the dramatic shifts of wealth the aggregate investment does not react much more differently than it would under the homogeneous beliefs. The main difference is during the two contraction periods. When beliefs are diverse the investment declines by more and the recovery pace is slower. A deeper slump can be attributed to the fact that the pessimists gain a larger weight during recessions. A more protracted recovery is due to the fact that the optimists accumulate wealth slowly and so the market is for long dominated by pessimists.

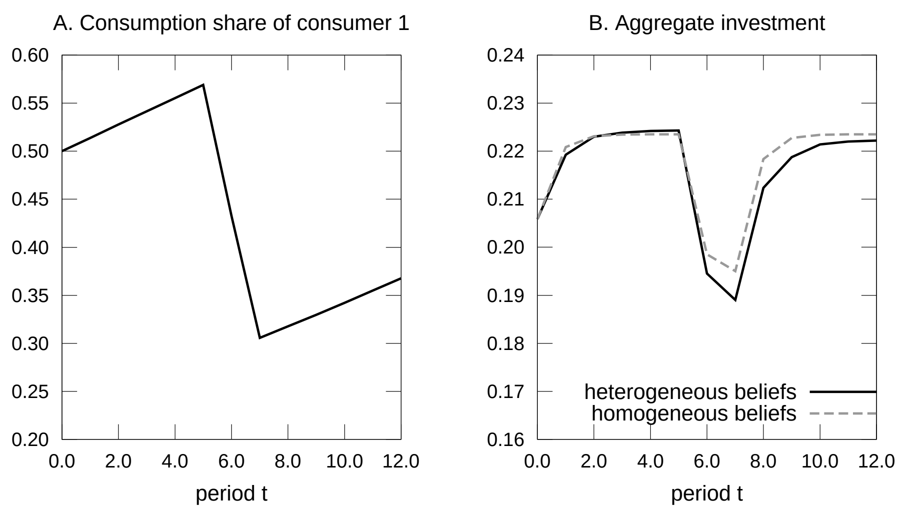

Figure 3: Consumption share and aggregate investment dynamics along the path $E, E, E, E, E, C, C, E, ...$

4.2 Regulated economy

Consider now imposing a financial constraint of the following form:

$$R_{t+1}(\sigma)k^i_{t+1}(\sigma) + b^i_{t+1}(\sigma) \geqslant -\kappa W_{t+1}(\sigma) \tag{20}$$

It states that a consumer's portfolio must not generate losses that are more than κ times the wage income in that period. That is the agent cannot take a "too" negative position in claims to equity or bonds. This prevents consumers from losing all their wealth, thus insuring that individual consumption is bounded away from zero. This immediately constrains speculation and stabilizes individual wealth shares. As a result consumption and investment also become less volatile. The wealth is distributed more evenly in the economy, see figure 4, and that is when speculation motives are the strongest. So, why does consumption become less volatile? The answer is that the presence of the financial constraints increases the downside of any risky bet. If the consumer turns out to be incorrect he becomes more likely to face the short-sale constraint. This reduces his incentives to speculate. Table 1 reports the welfare gains relative to the complete markets case. As κ approaches zero the welfare gain levels off. Because even small position in equity are sufficient to hedge the productivity risk it is optimal to ban any short-selling.

κ	10	6	4	2	1	0
CEV,%	0.09	0.81	1.11	1.27	1.34	1.39

Table 1:

Table 2 presents selected moments for the economy described above. I consider two choices of the borrowing limit: $\kappa = 1$ and $\kappa = 8$, the latter being effectively an unconstrained economy. Tightening of the borrowing limit drives volatility out of consumption and into asset prices. Welfare improves substantially and is equivalent to a 3.7% permanent increase in consumption. This suggests that a goal of financial market stability may conflict with social welfare maximization. Notice also that the change in the moments of the aggregate variables is negligible. This fact is especially important for investment that is much less volatile in the data than an RBC model predicts.

	ω^2	c^2	$ln(q^b)$	$ln(q^e)$	C	I
$\kappa = 1$	0.435	0.478	-0.037	3.148	1.072	0.357
	(0.120)	(0.040)	(0.008)	(0.031)	(0.011)	(0.030)
$\kappa = 8$	0.232	0.271	-0.041	3.097	1.069	0.355
	(0.201)	(0.091)	(0.002)	(0.023)	(0.012)	(0.032)

Table 2: Mean and standard deviation (in parentheses)

It is important to point out that the financial constraint is unlikely to significantly restrict risk-sharing capabilities. The reason is that it remains slack most of the time and activates only when the wealth distribution deviates sufficiently from the even allocation. This is unlike the Tobin tax that would affect the trade all the time. Yet, the Tobin tax may have a similar end result for it would decrease wealth transfers thereby reducing the frequency with which consumers could be impoverished.

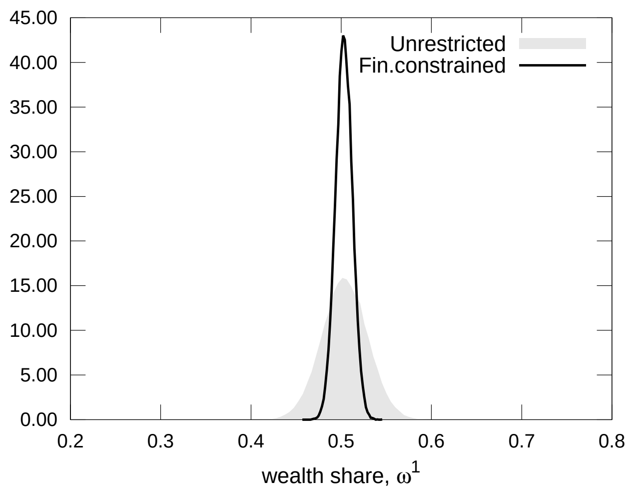

Figure 4: Ergodic distribution of the wealth share ω

5 Conclusions

This paper studies an RBC model with heterogeneous agents who disagree about the evolution of the economy's state. Disagreement leads to speculation, depressed investment, and increased volatility of consumption. The latter is a driving force behind a greatly reduced ex-ante welfare. Installing financial restrictions that hamper speculation reduces consumption volatility. But the same frictions could also reduce investment activity in the economy. Our simulations show that this is not the case and investment, in fact, increases marginally. At any rate the impact of belief heterogeneity and of financial restrictions on investment is small. Welfare losses in the unregulated setting stem mainly from increased consumption volatility. Financial restrictions counter speculative forces with ease and allow restoring the economy's welfare.

This work ignored idiosyncratic risk limiting the potential benefits that the unrestricted markets could have. This is an important venue for future research. It would also be valuable to understand what are the implications of disagreement about idiosyncratic states.

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