Capital Flows, Speculation, and Capital Controls*

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Abstract

Addressing policymakers' concerns that the post-GFC international financial flows to emerging economies have been speculative, we build a model with heterogeneous beliefs and use it to analyze different forms of capital controls. Motivated by speculative profits, the investors in our model purchase riskier portfolios and generate large and volatile capital flows. Wealth is "sticky" because investors withdraw from the market after large losses. This channel magnifies the welfare cost born by speculation.

We find that A) welfare gains from imposing capital controls can be substantial, equivalent to about 80 times the cost of business cycles, B) capital flow limits distort decisions only during large outflows or inflows, and they are preferable to the often-used transaction tax that is always active, C) controls may lead to increased volatility in the domestic financial markets. The calibrated model matches well Brazil's volatile capital flows and exchange rate and their response to a transaction tax.

Keywords: capital controls, speculation, international portfolios, asset pricing

JEL codes: F32, F38, D53, D84, G11, G12

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1 Introduction

Recent episodes of rapid capital inflows and exchange rate appreciation and subsequent reversals have led governments in emerging market economies to impose capital controls and to contemplate other forms of intervention. Government officials declared that these flows were speculative and thus inefficient. Speculative flows were feared to lead to loss of competitiveness, overheating, and propelling of fragile external liability configurations. Brazil, being affected the most, experimented with capital controls more than any other economy and achieved partial success according to Chamon and Garcia (2014).

The volatility of capital flows to emerging economies threatened the fragile recovery from the global financial crisis (GFC). This fear prompted the development of theoretical foundations for policy tools that would stabilize the flows; see the reviews by Korinek (2011), Jeanne (2013), and Rey (2013). The IMF, a long-time opponent of capital controls, changed its position and agreed to add capital controls to the macroeconomic policy toolkit for economies that risk overheating.[1]

The existing justifications, however, are motivated by the pecuniary externalities rather than the speculation which seems to be the primary motivation of policymakers.[2] We build the model that features heterogeneous beliefs in the spirit of Harrison and Kreps (1978), which enables us to model speculative flows. One could think of our model as of the U.S. financial sector betting on an emerging economy like Brasil. *Regardless of a particular belief configuration*, investors in our model assume large gross portfolio positions. The reason is that an asset price reflects the wealth-weighted average belief. So, irrespectively of the truth, investors are split into relative optimists and pessimists. A relatively optimistic investor buys what she thinks is an undervalued asset, and similarly, a relatively pessimistic investor sells what she thinks is an overvalued asset. Both optimistic and pessimistic in-

[1]The additional qualifications ensure that the risk is not self-inflicted by an undervalued currency and that the country has sufficient reserves to execute such policies.

[2]Speculation seems to contend policymakers of both emerging and small advanced economies. For example, Central Bank of Brazil (2016) states that one of its functions is to "to reduce volatility and speculation in the exchange market". Similarly, the governor of the Reserve Bank of New Zealand in 2000 stated that fundamentals could not constitute the complete explanation of [the NZD fluctuations against the USD]. See "The fall of the New Zealand dollar - why has it happened, and what does it mean?" available at http://www.bis.org/review/r001012b.pdf.)

vestors earn substantial speculative profits in cases when their predictions turn out to be correct. These profits motivate investors to accept volatile consumption, which entails increased volatility of assets prices, including the exchange rate which is a one-period bet on the relative value of the domestic basket of goods. Every period, wealth of the investors whose predictions turn out correct increases, or equivalently, their country experiences capital inflow. The capital inflows will reverse when the country's investors run out of luck,[3] leading to large swings in the exchange rate and asset prices. Movements of the net foreign asset position (NFA) – that would be constant had beliefs were homogeneous – is the key endogenous state that drives the above phenomena. That is, a model with heterogeneous beliefs has the potential to match key facts describing the emerging market economies. At the same time, speculation generates large welfare losses. First, because the investors willingly accept volatile consumption expecting to be compensated by future speculative profits. But there is a new second channel. Net foreign wealth is endogenously highly persistent when one of the countries looses a substantial portion of it. This happens because the investors that lost wealth have less funds to speculate with. Additionally, the investors that gain wealth have less incentives to speculate as asset prices approach their subjective valuation. So, extreme deviations from an equal distribution of wealth between countries may persist for a long time.

To justify capital controls we need to decide on a social welfare function. This is not trivial since the disagreeing agents perceive optimality differently. We follow Blume et al. (2018) because using their criterion is equivalent to assuming that policymakers are uncertain about the nature of the flows – fundamental or speculative – which is consistent with the motivation of our work.[4] This approach also ensures that our predictions hold for a range of beliefs rather than a particular configuration. Thus, the only degree of freedom in the model is the extent of potential disagreement that a planner is willing to consider.[5]

Brunnermeier et al. (2014) and Gilboa et al. (2014) offer alternative criteria. These criteria are subject to spurious anonimity, defined by Mongin

[3] An investor "runs out of luck" when the state that she is pessimistic about realizes. In this case, the investor's relative wealth decreases, assuming that her income volatility is not "too large".

[4] Additionally, the authors analyze a dynamic economy, and the provided welfare criterion is more suitable for quantitative work.

[5] Our argument holds even when the potential for disagreement is low, but the quantitative gains from various regulations are smaller.

(2005), when all agents agree on a suboptimal choice because their beliefs are wrong. The welfare comparisons based on these criteria are also massively incomplete rendering the work like ours impossible.

The "speculation" argument for regulation applies to any financial market. But, we believe that it is more compelling in the case of the international capital flows to emerging economies. First, the analysis of the global financial flows to emerging economies by the Global Financial Stability Report (2014) shows that macroeconomic fundamentals were unlikely drivers of these flows. According to the study, the most important determinant seems to be the global risk aversion measured by the VIX, a CBOE measure of financial market volatility in the U.S. [6] The same study finds these flows to be increasingly dominated by global mutual funds that are sensitive to global financial conditions, and, we believe, also sentiment. The economists behind the Global Financial Stability Report (2014) acknowledged that "many of the small investors putting money into mutual funds are less informed savers, who may panic-sell at signs of volatility."

Second, emerging economies have less data and it is of lower quality.[7] The poor quality of the national accounts data was pointed out by the IMF and market commentators numerous times.[8] In the next section, we report that the revisions to GDP growth series are substantially higher in emerging economies. Further, the errors are strongly and negatively related to the country size; see figure 1.[9]

Finally, emerging economies are more vulnerable as their financial markets are less developed and less liquid. So, large capital inflows or outflows

[6] Ahmed and Zlate (2014) argued that the growth differential between the developed and emerging economies affected capital flows to emerging economies. However, it is possible that the causal link is reversed: i.e., capital outflows stifle growth in emerging economies.

[7] The same argument was used by Morris (1996) to justify heterogeneous priors in the IPO market.

[8] E.g., see the "Country notes" in the World Economic Outlook (2016) and "Don't Lie to Me, Argentina" article in the February 25, 2012 issue of the Economist. See also table G in the World Economic Outlook for the system of national accounts used by each country. It shows that the national reporting standards are still largely unsynchronized.

[9] The analysis uses quarterly data from OECD.Stats and it spans the period 1980-2016. We used MSCI's classification of emerging economies provided, which is used by the financial professionals and which considers general quality of domestic financial markets in addition to income per capita. The GDP level used as an explanatory variable was measured as of the fourth quarter of 2010, the onset of the post-GFC capital inflows to emerging economies. All relations are found to be statistically significant at the 1% level.

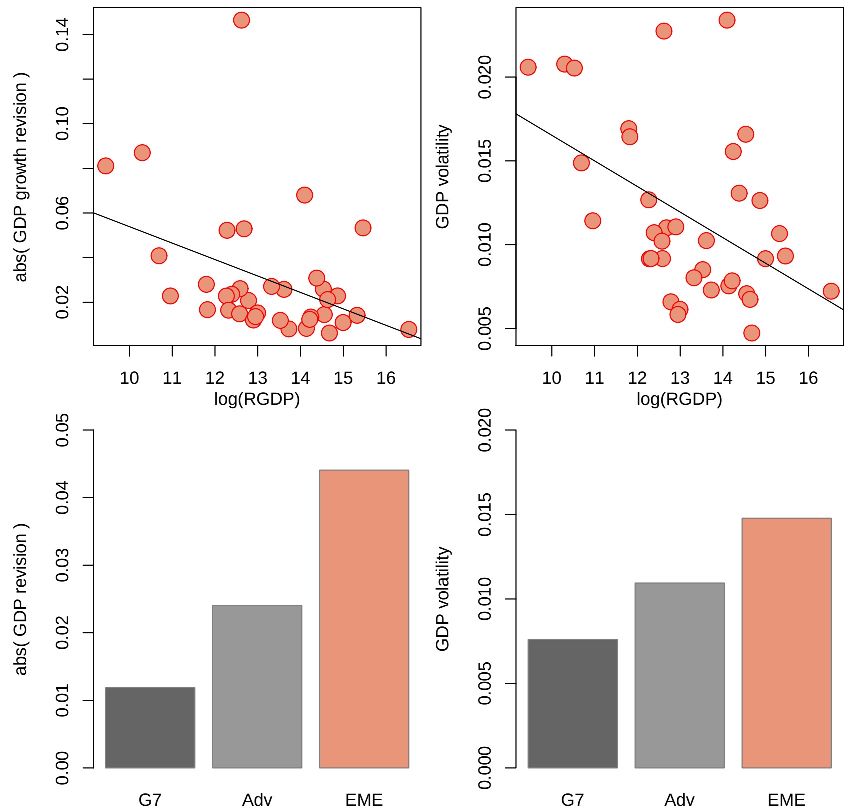

Figure 1: GDP growth revisions and GDP volatility in emerging and advanced economies

can have a substantial destabilizing effect on the recipient's financial system and its economy. This fact also implies that foreign investors wield disproportionate influence on asset prices. Together these features single out emerging economies as likely targets of financial speculation.[10]

The economic mechanism of our model prompts policies that curb extreme inflows rather than tax all foreign transactions. In particular, the cost of speculation comes in the form of volatile wealth distribution with a substantial tail "stickiness." With evenly-distributed wealth speculation

[10]Our analysis also applies to markets for newly issued stocks. The company data is typically limited and sometimes not well-understood, prices are more volatile, and the buyers are few when a firm's shares begin to trade.

is prevalent, and there are large and frequent wealth transfers. However, these wealth transfers have little welfare impact as the effect of consumption volatility is generally of second-order importance. But, if one economy accumulates a large wealth share, speculation slows, and capital flows (NFA fluctuations) fade. Dissipation of financial trade occurs because the wealthier economy has a disproportionate effect on asset prices pushing the latter closer to what is country's subjective valuation. The close alignment between the market price and the wealthy economy's subjective valuation reduces the latter's incentives speculate, and thus capital flows come to a halt. Those who lost wealth believe that there is room for arbitrage as prices move away from their subjective valuations. But poor economies cannot take large positions and, moreover, they must exercise caution not to lose more wealth. That is as financial luck rewards some market participants, the world economy looks more like the homogeneous beliefs economy in which the relative wealth positions persist over time. If someone suddenly revealed the true process and speculation stopped then some countries would get permanently stuck in poverty. Friedman (1953) foresaw that agents that are better able to predict the economic environment would prosper and the rest would be driven out of markets. However, the transitional dynamics may be erratic, and often those with less-accurate beliefs may temporarily dominate the market. This turbulence is the ultimate cost of speculation. Different capital controls could limit the tail behavior of the wealth distribution and bring about substantial welfare gains equivalent to permanently increasing consumption by up to 4%, 80 times larger than the cost of business cycles in the same setting. Our estimates show that only a third is due to increased volatility caused by speculation.

According to our analysis, an effective regulation must preclude countries from accumulating large external wealth positions, i.e., it must restrict the tails of the world wealth distribution. Large wealth inequality between countries can be prevented by imposing capital flow limits or a transaction tax. A transaction tax does not prevent large losses of net foreign wealth, but it reduces flows. It also distorts the economy at all times, and it could hamper price discovery as argued by Calvo and Mendoza (2000).[11] Capital outflow limits are less distortive: our quantitative simulations show that the economy's dynamics change only after large capital outflows/inflows occur, i.e., near the tails of the NFA distribution. Additionally, in practice, capital

[11] The authors show that transaction tax reduces incentives to gather information, and this could lead to herding and incomplete risk-sharing.

controls are imposed after large capital outflows occurred. In this case, the capital outflow limit is preferable because it prevents the financial position of the affected economy from worsening. The foreign transaction tax could not achieve this, unless it shuts down all financial trade. Moreover, it would reduce speed at which the economies could revert to "normal" conditions.

The rest of the paper starts with a description of the key results on survival in financial markets with heterogeneous beliefs and a brief overview of other work on capital controls. In section 2 we describe the model and the key economic forces. Section 3 presents a motivating example and section 4 states the key theoretical result. Section 5 analyzes several alternative forms of controls. Section 6.1 analyzes the model calibrated to Brazil. We conclude by proposing avenues for future work.

1.1 On True Beliefs

When analyzing disagreement it is convenient to think of a single dimension. For example, investors could disagree about the recession likelihood in an emerging economy. One immediately imagines that the investors are on different sides of the truth. In this case, shouldn't the planner use the average belief? Surely it would be closer to the truth than any of the individual beliefs. But, if the disagreement were along multiple dimensions using the average belief would not be compelling. For example, the foreign investors could be optimistic about an emerging economy's growth, and the domestic investors in that emerging economy could be pessimistic about the world commodity prices. The average belief is incorrect about both growth and the commodity prices: it is optimistic in the first dimension and pessimistic in the second. Our work does not attempt to inform what beliefs should be used by the planner. In the spirit of robustness, our work is based on the premise that the planner should not commit to any particular truth.

The truth could be learned by observing market outcomes, e.g., security prices. We do not allow learning from prices and assume that there is no useful information to be extracted from the investors' beliefs. Financial trades are anonymous in reality, in which case eliciting the investors's private information is impossible. If the investors' identities were publicly observed, and in real time, learning is possible only if they rely on the same model. Learning is also complicated even theoretically when the object to be learned in infinite-dimensional, e.g., the forecast distribution of an emerging economy's growth rate. We believe these are sufficient reasons to rule out learning.

1.2 Key results from survival literature

It is instructive to introduce several key results from the literature on heterogeneous beliefs that this work builds on. All of these results were developed in the context of bounded endowment economies with complete, that is unregulated, financial markets. It is also crucial to point out that all information is common. That is agents understand that others have different beliefs which constitutes a departure from the rational expectations paradigm, an issue that we discuss later. In this setting, with complete financial markets and heterogeneous beliefs, any agent who has less-accurate information, as measured by relative entropy, is going to be driven out of the market. That is his consumption converges to zero on all paths of events except for a small set that has zero probability. This result was shown by Blume and Easley (2006) and, in a different form, in Sandroni (2001). Blume and Easley (2009) show that if all agents have equally accurate information then each agent will infinitely often have consumption that is arbitrarily close to zero. In either case, if the period utilities are unbounded below the agents will certainly experience a dismal flow of utility. In section 4 we show that the same analysis applies in the multi-good environments. Regulated financial markets are desirable because they do not allow for the full effect of survival forces. But belief heterogeneity *per se* does not mandate regulation of financial markets as the competitive equilibrium allocation is *subjective*-Pareto optimal after all. But as argued in Blume et al. (2018) the subjective Pareto criterion may lead to unreasonable social choices in environments with heterogeneous beliefs. Similar arguments are made by Brunnermeier et al. (2014) and Gilboa et al. (2014).

That is where the welfare criterion becomes important. Following Blume et al. (2018) we assume that neither the planner-regulator nor any agent knows the true evolution of the world. Every agent believes that he has the most accurate information. Everyone's belief is publicly known. But it is impossible to determine with certainty whose beliefs are more accurate for it is impossible to separate the effect of luck. If the planner-regulator knew the true process he should share this information with all the agents. The decision about the set of restrictions on financial markets has to be made without knowledge of the true evolution of the world. All that is known is that the true process comes from the same pre-specified set as the agents' beliefs. That is the set of financial restrictions is chosen behind the veil of ignorance: never knowing the truth. This argument introduces a new degree of freedom: the set of admissible beliefs. The larger it is the stronger disagreement between agents can be and, therefore, the faster agents can loose their wealth. So, more disagreement provides more incentives to regulate financial markets. We discuss ways to choose this set in section 5.

The survival forces borne in the environments with disagreement are consistent with many realistic phenomena. The most important are capital flow and exchange rate volatility. In our model the home equity bias co-exists with large and volatile capital flows. At the same time observed exchange rates are very volatile despite a relatively stable supply of goods. In contrast, with homogeneous beliefs and complete financial markets the real exchange rates are determined by the relative supply of goods and are generally smooth. In our model, exchange rates are affected by belief heterogeneity directly via their influence on the equilibrium pricing kernel and indirectly via their effect on the world's wealth distribution. The model that we analyze is suitable for policy analysis and we also demonstrate that it matches well the experience of Brazil.

Belief disagreement is observable to some degree. One possibility is to compare published forecasts of various institutions. For example, on December 7, 2014 Agence France-Presse reported that Troika considered the 2.9% forecast growth too optimistic and predicted a 3% budget deficit against the Greek's estimate of 0.2%. On September 22, 2014 Brazil's central bank released its GDP forecast of 0.7% and two days later Morgan Stanley based in the U.S. released their own forecast of 1.0%. However, we are not aware of any systematic data analysis. Another possibility is to study professional forecast surveys. Unfortunately, we do know of any (publicly available) data source that would allow us to make an international comparison. Disagreement can be gauged indirectly from trades between market participants. Consider an environment with income realizations that are independent across time. In this case, if the beliefs were homogeneous, the maximum trade that one could observe in equilibrium would be bounded above by the size of the income support. Any trade that is larger in size must be speculative. We take this path in a different work. Here, we only demonstrate that a failure to recognize belief heterogeneity may lead to substantial welfare losses. For our calculations we bound belief differences by the statistical uncertainty that is present in a typical macroeconomic series.

1.3 Related work

From the modeling point of view the closest papers are Albuquerque et al. (2007) and Dumas et al. (2014). They too analyze endowment economies with heterogeneous beliefs. The model in Albuquerque et al. (2007) explains several important facts about the U.S. capital flows: two-way flows, momentum in equity positions, and return-chasing. The key element of this model is heterogeneous financial sophistication of investors in each country. The more-sophisticated agents take a different position in foreign assets than the less-sophisticated ones, and this generates two-way capital flows. Momentum and return chasing are driven by an endogenous level of investment in local equity markets by the more-sophisticated agents. Dumas et al. (2014) address a different set of facts: co-movement of returns and international capital flows, home equity bias, dependence of firm returns on home and foreign factors; and abnormal returns around foreign firm cross-listing in the home market. Both papers analyze one-good settings, ignoring fluctuations in the exchange rate that is a major factor determining capital flows in our view. Neither of these papers considers regulation of capital flows, which is the main objective of this paper.

Other justifications of capital controls in the literature are based either on pecuniary, as in, e.g., Jeanne and Korinek (2010) and Bianchi and Bengui (2011), or demand externalities, as in Farhi and Werning (2013). Like Farhi and Werning (2013) we study complete financial markets. But our setting could be viewed through the lens of missing markets, very much like in models with overlapping generations (OG). While in an OG setting unborn agents are excluded from the financial markets, in our work agents cannot insure against a possibility of being born with inaccurate beliefs. This market is missing because neither planner nor any other agent in the economy can testify to any belief's accuracy. So, it is a model of capital controls under imperfect information. Our model features a novel for this type of models mechanism – survival. As long as there is disagreement, all or some agents, by poor luck or by an inability to process financial information, will lose substantial amounts of wealth in financial markets. The survival mechanism may lead to unexpected consequences if there are other frictions in the model as in Cogley et al. (2014). In this way, this work is complementary to the vast literature on macro-prudential policy.

Finally, Calvo and Mendoza (2000) find another risk from unregulated financial markets. They show that financial globalization may weaken the incentives to gather costly information and may promote herding behavior.

This would rationalize belief heterogeneity in our model and it would solidify our results.

2 The Model

Time is discrete and indexed by $t = 0, 1, 2, ...$ The exogenous state of the economy z_t is a first-order Markov process with finitely many states, $\mathcal{Z} = \{1, ..., S\}$, and a probability transition matrix Π^0. Initial state z_0 is given. A partial history of the state realizations $(z_0, ..., z_t)$ is denoted by z^t and its probability by $\pi(z^t|z_0)$.

There are two countries, each is populated by a representative consumer-investor.

There are two perishable goods traded every period. Country i produces good i that is traded at price $p_i(z^t)$.

Financial markets are dynamically complete. In each date and history financial markets trade S Arrow securities. An Arrow security j that is purchased in period t pays one unit of account in period $t+1$ if state $z_{t+1} = j$ realizes. The price of Arrow security j is denoted by $Q_j(z^t)$.

The portfolio of Arrow securities purchased by country i is denoted by $a^i(z^t) \equiv (a^i_1(z^t), ..., a^i_S(z^t))$. The initial distribution of financial wealth $(a^1(z_0), a^2(z_0))$ is given.

Household in country i trades in financial and goods markets to maximize the expected life-time utility given by:

$$E^i \left[\sum_{t=0}^{\infty} \beta^t u(g^i(c_1(z^t), c_2(z^t))) \Big| z_0 \right], \ \beta \in [0, 1). \tag{1}$$

Expectations of investors in country i are computed using their subjective probability measures $\pi^i_t(z^t|z_0)$ which could differ from the truth $\pi^0(z^t|z_0)$. The subjective probability measures could vary over time, and this enables us to consider learning from exogenous events. However, this is inconsequential to our key insights, and we introduce learning only in the calibrated model in section 6.

Function g^i is a constant return to scale (CRS) consumption aggregator. We assume that households' display *consumption home bias*. Hence, consumption spending in country i is biased towards the domestically produced good i. In the case of the constant elasticity of substitution (CES) aggregator, this assumption is isomorphic to assuming that trading is subject to a proportional cost that is rebated back to consumers.

Assumption 1.
$u(c)' > 0, u(c)'' < 0, \quad \forall c > 0.$

Household in country i receives $e_i(z_t)$ units of domestic good i.

Assumption 2.
There exists $\underline{e} > 0$ *such that* $e_i(z) > \underline{e}, \quad \forall z \in \mathcal{Z}, i \in \{1, 2\}$.

The above assumption guarantees that the utility in financial autarky, defined later, is always bounded below.

Budget constraint of a household living in country i after history z^t is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + \sum_j Q_j(z^t)a_j^i(z^t) = I^i(z^t), \tag{2}$$

where $I^i(z^t)$ is "cash-in-hand" that consists of his non-financial income e_i and the market value of his financial possessions:

$$I^i(z^t) \equiv p_i(z^t)e_i(z^t) + a_{z_t}^i(z^{t-1}). \tag{3}$$

A *competitive equilibrium* is a price system $\mathcal{P} = \{p_1(z^t), p_2(z^t), (Q_j(z^t))_{j=1}^S) : \forall z^t\}$, an allocation $\mathcal{C} = \{(c_1^i(z^t), c_2^i(z^t))_{i=1}^2 : \forall z^t\}$, and a security trading plan $\mathcal{A} = (a^1(z^t), a_2^i(z^t))_{i=1}^2 : \forall z^t\}$ such that:

1. given the price system $\mathcal{P}$, the allocation $\mathcal{C}$ and the security trading plan $\mathcal{A}$ solve each household's optimization problem;

2. financial and goods markets clear: $\forall z^t, j = 1, 2$,

$$c_j^1(z^t) + c_j^2(z^t) = e_j(z^t) \tag{4a}$$
$$a_j^1(z^t) + a_j^2(z^t) = 0. \tag{4b}$$

3 An Example Economy

This section sets up a motivating numerical example of an unregulated economy. We study three belief specifications to illustrate the forces shaping the dynamics of wealth of the two countries. In the first setting both countries hold correct beliefs. It is a useful benchmark. In the second setting both countries hold equally-incorrect beliefs. This setting highlights speculative forces leading to large and volatile capital flows. In the third setting only one country holds correct beliefs. This setting demonstrates the survival force that drives the country with inaccurate beliefs towards financial extinction. To make the analysis simpler we assume that the two economies are symmetric under the true data generating process in all cases.

We concentrate our attention on the dynamics of net foreign wealth (NFW), real exchange rate (RER), and consumption (C). In appendix E we describe how welfare changes over a large set of possible beliefs.

3.1 Functional forms and parameter assumptions

For the analysis in this section and other numerical examples in other sections we assume a CRRA utility function and a consumption CES aggregator:

$$u(c) = c^{1-\gamma}/(1-\gamma), \quad \gamma > 0 \tag{5a}$$

$$g^1(c_1, c_2) = (sc_1^\rho + (1-s)c_2^\rho)^{1/\rho}, \quad s \in [0.5, 1], \rho \leqslant 1 \tag{5b}$$

$$g^2(c_1, c_2) = ((1-s)c_1^\rho + sc_2^\rho)^{1/\rho} \tag{5c}$$

where ε denotes the elasticity of substitution (ES) between the two goods. Unless stated otherwise we assume the following preference parameters in our numerical examples:

$$\beta = 0.96, \quad \gamma = 2, \quad \rho = 0.8, \quad \sigma(e) = 0.05. \tag{6}$$

To simplify the discussion we assume that the exogenous state z_t is i.i.d. and use the following notation: $\pi^0 \equiv (prob(z_{t+1} = 1|z^t), ..., prob(z_{t+1} = S|z^t))$. Agents in country i also believe that z_t is an i.i.d. but assign different probabilities: $\pi^i \equiv (prob^i(z_{t+1} = 1|z^t), ..., prob^i(z_{t+1} = S|z^t))$.

The above imply the following aggregate price indices:

$$P^1 = (s^\varepsilon p_1^{1-\varepsilon} + (1-s)^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}, \tag{7a}$$

$$P^2 = ((1-s)^\varepsilon p_1^{1-\varepsilon} + s^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}. \tag{7b}$$

Let $q \equiv p_1/p_2$ and $Q \equiv P^1/P^2$ denote terms of trade and real exchange rate, respectively.

3.2 Both countries hold correct beliefs

In this case the equilibrium allocation and the price system are:

$$(c_1^1(z^t), c_2^1(z^t)) = (fe_1(z_t), (1-f)e_2(z_t)) \tag{8a}$$
$$(c_1^2(z^t), c_2^2(z^t)) = ((1-f)e_1(z_t), fe_2(z_t)) \tag{8b}$$

where

$$f \equiv \frac{s^\varepsilon}{s^\varepsilon + (1-s)^\varepsilon}.$$

Several important observations can be made. Because markets are complete the CE allocation is strongly stationary, namely it is a function of the exogenous state only. Moreover, there is full risk-sharing and the countries consume constant fractions of the world supply of each good. But the countries' aggregate consumption levels are less volatile than their outputs. Net foreign wealth of each country changes, and so capital flows are non-zero, only if the exogenous state changes. Capital flow, when non-zero, equals 0.1% of country 1's GDP and NFW of country 1 fluctuates between -0.06% and +0.06% of GDP. Real exchange rate oscillates between 0.917 and 1.091, and its average value remains constant at 1.

Welfare of each country equals -1.5936. To set a comparison benchmark, we compute cost of business-cycle fluctuations as in Lucas (2003). It is defined as a welfare effect of removing output volatility.[12] If output were fixed at its expected level in each country then welfare of each economy would be -1.5908. This constitutes a gain that is equivalent to permanently increasing consumption by 0.2% and it is of a similar magnitude as found by Lucas (1978). We will use this magnitude as a benchmark for potential welfare losses or gains that are typically obtained in endowment economies like the one considered here.

[12] We define the welfare cost as the effect of income rather than consumption volatility. This is equivalent to the original definition because the consumption and income processes coincide in Lucas (2003).

	A. correct,correct	B. wrong,wrong	C. wrong,correct
NFW	0.000	-0.306	-0.813
	[0.000,-0.000,0.000]	[1.301,-5.496, 1.974]	[1.331,-5.753, 1.775]
RER	1.000	1.000	0.913
	[0.087, 0.917,1.091]	[0.259, 0.483, 2.070]	[0.214, 0.474, 1.835]
Consumption	0.800	0.781	0.744
	[0.036, 0.764,0.836]	[0.102, 0.418, 1.006]	[0.112, 0.363, 0.992]

Table 1: Selected statistics for the numerical examples described in section 3. In square brackets we report standard deviation, minimum and maximum.

3.3 Both countries hold incorrect beliefs

We assume that beliefs are symmetric with respect to the truth, which remains the same:

$$\pi^1 = (0.525, 0.475), \quad \pi^2 = (0.475, 0.525), \quad \pi^0 = (0.50, 0.50).$$

Both economies hold equally-incorrect optimistic beliefs about their domestic output.[13] Figure 10 in appendix G plots key macroeconomic variables. The mean consumption, NFW, RER remain constant over time. At the same time, the support of the NFW expands and the unconditional, i.e., across paths, variance of the macroeconomic aggregates increase with time. That is, the symmetric belief heterogeneity affects volatility but not level of the quantities and prices.

Table 1 column B reports country 1's key economic statistics for the first 100 periods.[14] Volatility of each variable is several times larger than in the economy with homogeneous beliefs. Country 1's NFW varies between -550% and 197% of its GDP. RER varies between 0.483 and 2.070. The extreme variation of RER and the strong negative correlation with NFW is the reason for the average NFW/GDP to be negative and significant despite the mean NFW being zero.

The increase of volatility takes its toll on welfare. Country 1's true welfare is -1.6117 which should be compared to -1.5936 in the homogeneous beliefs case. Welfare in country 2 is the same as that of country 1 by symmetry. The loss of utility in the case with heterogeneous beliefs is equivalent

[13] It is inconsequential if the agents are optimistic or pessimistic.

[14] It takes around 100 periods for the economy to visit most of the NFW's distribution support.

to a permanent reduction in consumption of 1.2%, more than 6 times larger than the cost of business cycle fluctuations. It is caused by speculative motives: investors think that they have more accurate beliefs than the rest and along some paths it may look like their strategies pay off. But fewer paths lead to wealth accumulation than agents expect.

Large portion of the welfare loss stems from the fact that NFW becomes "sticky" when it deviates far from 0. When country 1's NFW is zero, that is financial wealth is distributed evenly, speculative trading is at its highest level. When country 1 accumulates wealth it commands a stronger influence on asset prices and valuations shift towards the level implied by its beliefs. For this reason country 1's incentives to engage in speculative trading decline. Country 2's speculative motives strengthen as asset prices deviate from what it considers to be a fair evaluation. However, country 2's wealth declines and, hence, its ability to invest and speculate. The overall result is a decline in speculative trading and, hence, a decline in wealth fluctuations. That is agents protect themselves from exposure to large financial losses by taking smaller positions. But this also limits the upside potential and reclaiming lost wealth now requires more time.

Interestingly, learning could amplify the "sticky wealth" effect. Assuming that the truth could be learned, the beliefs of learning agents converge over time and speculation dissipates. Consider then an agent who makes a series of unsuccessful speculative trades. Her wealth declines while her and others' incentives to speculate decline. As a result, the wealth distribution would become "sticky," but in this case because the agents would stop to disagree.

3.4 Only one country holds correct beliefs

In the final configuration we assume that only country 2 has correct beliefs:

$$\pi^1 = (0.545, 0.455), \quad \pi^2 = \pi^0 = (0.5, 0.5).$$

With these beliefs the consumption volatility is the same as in the previous example. Investors in country 1 are optimistic about their domestic output, but again this fact is inconsequential. Table 1 column C reports country 1's key economic statistics for the 100 period simulation. Because country 1 has less accurate beliefs its financial wealth trends down, the result of survival forces described in Sandroni (2000) and Blume and Easley (2006).[15]

[15] Investors with more accurate beliefs prosper and survive and those with less accurate beliefs lose wealth and eventually are driven out of the market. Friedman (1953) predicted

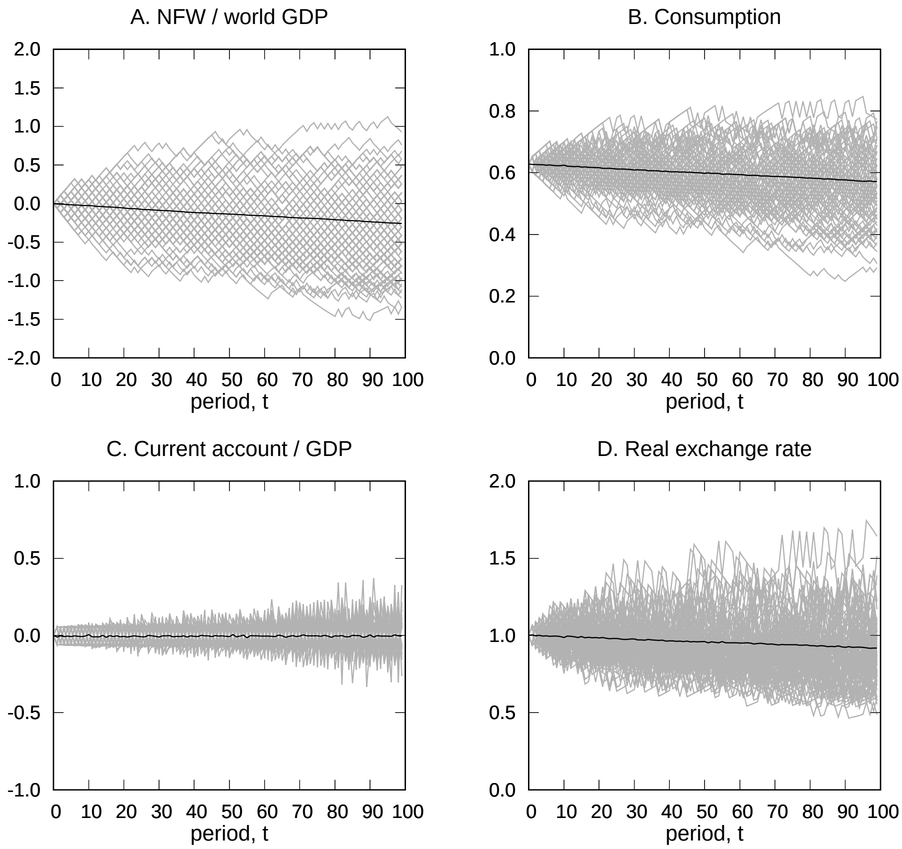

Figure 2: Dynamics of country 1 endowed with less-accurate beliefs. Grey lines represent 100 random paths, black lines represent the averages across paths.

The lower average NFW explains why the mean consumption, NFW, RER are lower than in column B. The volatility of NFW remains essentially unchanged and that of RER declines 17% relative to column B. Volatility of consumption remains the same by design. Country 1's NFW worst and best case scenarios worsen. Real exchange rate reacts similarly. Country 1's true welfare is -1.6383. The loss relative to the homogeneous beliefs case is equivalent to permanently decreasing consumption by 2.8%. This welfare effect

such dynamics and referred to it as "survival".

is 16 times larger than the cost of business cycles in the same setting.[16]

Because consumption volatility is the same in this and the previous setting, we can compute the effect of the survival forces on welfare: it is equivalent to a 1.6% permanent loss of consumption. It is larger than, but comparable to, the 1.2% loss due to speculative volatility.

Figure 2 plots 100 sample paths of the key macroeconomic indicators for this economy. Panel A plots the key statistic: net foreign wealth position of country 1. Country 2's wealth is the negative of country 1's value. If both countries held the same beliefs wealth position of each country would remain zero along any path. Because country 1 has less accurate beliefs its wealth is drifting down on average. In period 100 the average path of NFW reaches -25.7% of the world GDP. More strikingly, wealth of country 1 is extremely variable. By period 20 it can be anywhere between -95% and 80% of the *world* GDP. The dynamics of wealth translates directly into that of consumption and real exchange rate, see panel B and D. As country 1 loses financial wealth its consumption drifts down and the real exchange rate depreciates. Current account, shown in panel C, grows more volatile over time and the absolute value of a period balance can reach 25% of domestic GDP. This extreme volatility has profound implications for welfare as the numbers presented above attest. The survival forces present in this example are at the heart of the arguments in the next section.

4 Theoretical Results

First, define welfare level of country i evaluated using belief π:

$$W^i_\pi(\pi^1, ..., \pi^I, M) = (1-\beta)\sum_{t=0}^{\infty}\sum_{z^t}\beta^t\pi(z^t)u(c^i(z^t)). \tag{9}$$

It depends on the probability measure used to compute the expected utility and on the allocation assigned to agent i. The allocation, being part of a competitive equilibrium, depends on beliefs of all agents $(\pi^1, ..., \pi^I)$ and on the financial market structure M. We use the utilitarian social welfare

[16]Different levels of belief accuracy create a potential for large welfare losses. However, this effect does not play a role in our argument. The discrepancy between beliefs, as measured by entropy, determines the extent of speculation and is the key element of our model. The lowest welfare in our numerical examples is achieved when investors have equally accurate but symmetrically opposite beliefs.

function:

$$W_{\pi^0} = \sum_i W^i_{\pi^0}.$$

Observe that individual welfare levels are evaluated using the objective probability distribution. As stated in the introduction this paternalistic welfare criterion is motivated in Blume et al. (2018).

When the financial markets are complete the CE allocation is subjective-Pareto optimal, where the adjective "subjective" emphasizes the fact that the criterion uses subjective beliefs to evaluate individual welfare. For this reason the marginal utilities must be equalized across countries. That is, using $\lambda^i > 0$ to denote Pareto weight of country i:

$$\frac{\lambda^1 \beta \pi^1(z^t) u'(c^1(z^t)) g_1(c^1_1(z^t), c^1_2(z^t))}{\lambda^2 \beta \pi^2(z^t) u'(c^2(z^t)) g_1(c^2_1(z^t), c^2_2(z^t))} = 1.$$

As trade in goods is frictionless we can use relation (13) derived in the appendix to get:

$$\frac{u'(c^1(z^t))}{u'(c^2(z^t))} \cdot \frac{1}{Q(z^t)} = const \times \frac{\pi^2(z^t)}{\pi^1(z^t)}, \tag{10}$$

where $Q(z^t) = p^1(z^t)/p^2(z^t)$ is the real exchange rate.

It is useful now to relate to the setting in which only one good is produced and consumed. In this case $Q(z^t) \equiv 1$ and behavior of the ratio of marginal utilities is determined solely by the likelihood ratio $\pi^2(z^t)/\pi^1(z^t)$. The analysis of Blume and Easley (2006) is based on this relation. The same analysis continues to apply as long as one can show that $Q(z^t)$ is bounded. This is indeed the case and we first present an intuitive explanation. Suppose that consumers in one of the countries are close to a financial ruin, i.e., $\limsup_{t\to\infty} c^1(z^t) = 0$. With only one country present in the market the relative price of goods is $g_1(e_1(z^t), e_2(z^t))/g_2(e_1(z^t), e_2(z^t))$, a well-defined finite value. Despite the total spending of country 1 approaching zero, the relative consumption of individual goods remains balanced. So, a possibility that one of the countries could vanish does not imply unbounded dynamics of the real exchange rate. The formal statement is given below and the proof is confined to appendix B.

Proposition 1. *The real exchange rate is bounded:*

$$\exists m > 0, M > m: \; Q(z^t) \in [m, M] \quad \forall t, z^t.$$

Because the real exchange rate is bounded it must converge to a constant as one of the countries is being driven to poverty. So, one can apply the same arguments as in Blume and Easley (2006) to show that the country with less accurate beliefs will be driven out of the market. We state this result without proof below for the case with $I = 2$.

Proposition 2. *The country with less-accurate beliefs is driven out of the market:*

$$
\begin{aligned}
E(\pi^1, \pi^0) > E(\pi^2, \pi^0) &\rightarrow \lim\sup_{t\to\infty} c^1(z^t) = 0, \quad \pi^0 - a.s., \\
E(\pi^1, \pi^0) = E(\pi^2, \pi^0) &\rightarrow \lim\inf_{t\to\infty} c^i(z^t) = 0, \quad \pi^0 - a.s., \quad i = 1, 2.
\end{aligned}
$$

Proposition 2 implies that when beliefs are heterogeneous countries opt for a more volatile consumption wrongfully expecting speculative financial gains. Leaving the speculative forces unrestricted has dismal implications for welfare. But, restricting financial markets means closing some insurance venues. The aim of any financial regulation is thus to balance speculation and insurance. To understand this consider the following case. Suppose that country 1 has less accurate beliefs and $\lim\sup_{t\to\infty} c^1(z^t) = 0, \pi^0 - a.s.$ Then for any $m > 0, p \in (0, 1)$ there exists $T > 0$ such that $prob(c^1(z^t) < m) \geqslant p, \forall t > T$. The maximum consumption of country 1 is also uniformly bounded above by some $M > 0$ because endowments are. So, the life-time utility is bounded above by $\beta^T u(M) + (1-\beta^T)u(m)$ that converges to $u(m)$ as β increases. Crucially, an increase in β does not affect T. Because m and p are arbitrary and the utility function is unbounded below the true welfare of country 1 can be arbitrarily low. In appendix D we show that the utility under the financial autarky is bounded below. So, when agents are sufficiently patient even the financial autarky can dominate the unrestricted financial markets. This proves the main result of this paper that is stated formally below.

Proposition 3. *Suppose that the period utility function is unbounded below and $\pi^1 \neq \pi^2$. If the agents are sufficiently patient then the true welfare under complete markets is lower than under financial autarky:*

$$
\exists \bar{\beta} : \forall \beta > \bar{\beta} \quad W_{\pi^0}(\pi^1, \pi^2, CM) < W_{\pi^0}(Aut)
$$

4.1 Strength of survival forces with multiple goods

In this section we ask the question of whether the survival forces are stronger or weaker in a multi-good environment. In a single-good environment their strength depends solely on the level of disagreement as measured by the entropy. With multiple goods, an endogenous response of the real exchange rate interferes with the survival process. This can be seen from equation (10). Suppose that country 1 has less accurate beliefs than country 2. So, country 1 must be loosing wealth on average and with wealth leaving its currency weakens. Keeping the relative likelihood π^1/π^2 fixed, as $Q(z^t)$ declines the relative consumption $c^1(z^t)/c^2(z^t)$ must increase according to (10). That is the endogenous response of the real exchange rate counteracts the survival forces. For a formal argument see appendix C.

There is another consideration. As the real exchange rate decreases country 1 must rely more heavily on consumption of the domestic good. The price incentives is one reason, but there is another – the natural borrowing limit of country 1 shrinks. So, it takes fewer negative shocks to exhaust the borrowing capacity in the multi-good setting than in the single-good setting. In the examples in section 3 countries can borrow up to approximately 2.5 times their GDP, while in the equivalent one good setting they would be able to borrow up to 20 times their GDP.

For the parametrizations that we consider the borrowing capacity effect dominates the RER effect. Hence, the survival forces are stronger in the multi-good model than they would be in a single-good model. But the reverse could also be true.

5 Capital controls

In this section we analyze several forms of capital controls and their effect on welfare. The first form of controls is a simple lower limit on the net foreign wealth (NFW) position. The second form of controls is a tax on foreign transactions. There is a profound difference between these two forms of controls. A limit on NFW position activates infrequently, but its impact on the allocation is significant when it does activate. A transaction tax while being always active has only a small effect on the allocation.

5.1 Lower bound on net foreign wealth

Consider the following restriction on financial trade:

$$a^i_j(z^t) \geqslant -B, \quad \forall j, z^t. \tag{11}$$

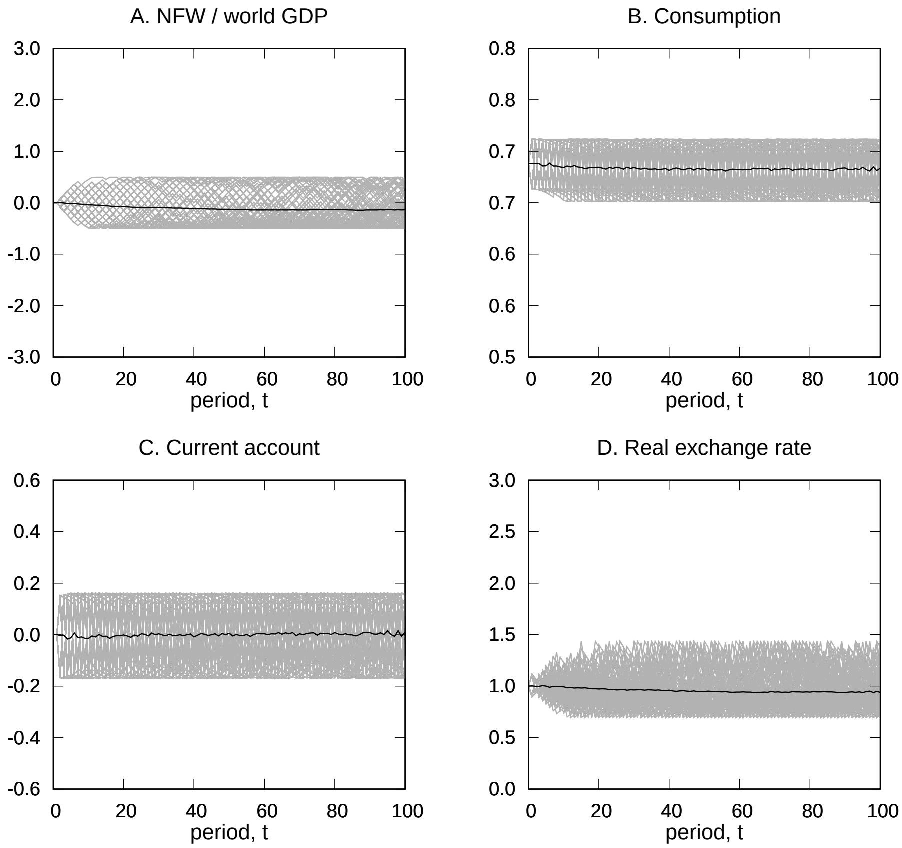

Figure 3: Dynamics of country 1 endowed with less-accurate beliefs. Grey lines represent 100 random paths, black lines are averages across paths. Financial trade is subject to an exogenous limit: B=0.5.

Figure 3 plots the key macroeconomic variables for economy 1 when both countries are subject to an exogenous lower bound on NFW position $B = 0.5$. Compared to figure 2 variability of all series is reduced significantly. Wealth, shown in panel A, is still trending down but now it never decreases below -50% of the world GDP. The average path of wealth ends at -13.5% of the world GDP as opposed to -42.3% when the financial markets are unrestricted. Similarly, the average path of consumption ends at 0.684 instead of 0.661, a 3.2% improvement. The average real exchange in period 100 is 6.4% stronger, settling at 0.939 instead of 0.883. As before consumption and the real exchange rate trend down but consumption does not decrease below 0.652 while before it could be as low as 0.436. The real exchange rate does not depreciate below 0.694 while before it could reach 0.321. It is the significant improvement in the worst case outcome that improves countries' welfare. The true welfare of country 1 and 2 is, respectively, -1.4812 and -1.4460. Compare these to -1.4991 and -1.4436 under the unrestricted financial markets. At the same time the limit is binding only 4.4 percent of time. The other time the financial constraint is inactive and neither risk-sharing nor speculation between consumers in different countries is restricted.

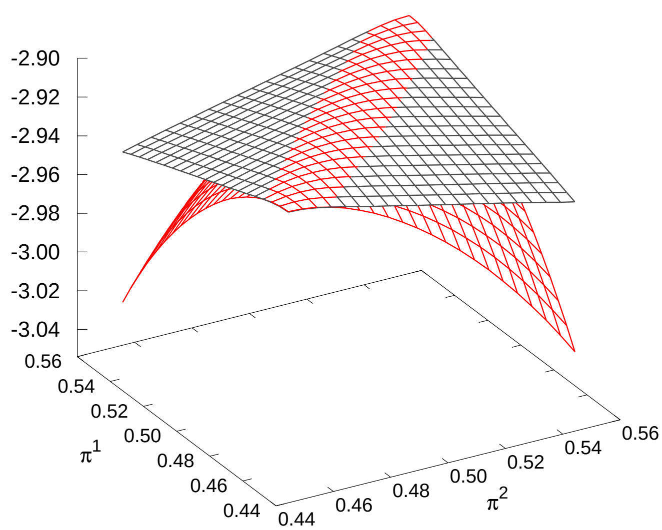

Figure 4: Welfare surfaces for the unregulated economy (red) and the economy with a net foreign wealth limit (black). Each point corresponds to a different admissible assignment of beliefs. Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

Controls?	Mean	Minimal
Unregulated	-2.932	-3.023
NFW limit	-2.922	-2.945
	(0.4%)	(2.6%)
Tobin tax	-2.923	-2.948
	(0.3%)	(2.5%)

Table 2: True welfare levels in different capital controls regimes. Values in brackets indicate the corresponding consumption equivalent variations. The NFW limit is $B = 0.5$; the transaction tax level is $\tau = 0.67\%$.

Figure 4 plots the average true welfare for a range of admissible beliefs: $(\pi_1^1, \pi_1^2) \in [0.45, 0.55]^2$. Consider the world without capital controls (red surface). Along and close to the diagonal where $\pi_1^1 = \pi_1^2$ welfare in the world is high. The reason is that motives for speculation are limited and consumption of each economy is relatively stable. The lowest welfare corresponds to the case when the countries hold maximally different beliefs, that is along the diagonal $\pi_1^1 = 1 - \pi_1^2$. The maximal loss of utility, relative to the homogeneous beliefs benchmark, is large and it is equivalent to a 3.8% permanent reduction of consumption. Turn to the economy with a limit on the NFW (black surface). We set the limit at $B = 0.5$ which is equal to an average value of a country's output. If beliefs were homogeneous the welfare would be nearly as high as in the unregulated economy. The reason is that when disagreement is small controls are inactive. So, if one believes that there is little disagreement imposing capital controls would have a negligible impact (loss) on utility. In fact, at the homogeneous-beliefs diagonal welfare loss is identically zero. At the same time there is a substantial gain to be made if disagreement turns out to be large: the maximal loss of utility is now equivalent to losing 1.1% of consumption. The welfare cost of unrestricted capital markets could be more substantial if the limit B were chosen optimally.

There are two ways to measure welfare in the two economies. First, we could average across all possible belief realizations in $[0.45, 0.55]^2$. Second, we could choose the minimal (worst-case) welfare across all possible beliefs. Blume et al (2015) discusses different choices and argues in favor of the second. We report both in table 2. According to the first measure (mean) imposing capital controls is equivalent to permanently increasing every country's consumption by 0.4%. This is a substantial effect given that endowments are stationary and that standard deviation of endowments is only 5%. According to the second measure (worst-case) welfare improvement is equivalent to a 2.6% permanent increase in consumption. To put it in perspective, the present discounted value of this windfall is valued at more than 50% of a country's GDP.

How does the NFW limit affect the policy functions? Obviously, the limit impacts the saving decisions. So, consumption must be affected also. But the real exchange rate is affected only indirectly. That is the real exchange rate as a function of country 1's wealth is the same in the two settings, but the dynamics of wealth itself is different. Figure 5 plots the dynamics of the NFW of country 1. First, discrepancy between the two paths of the NFW appears immediately. This difference is attributable to variable strength of precautionary motives. For the fear of being constrained consumers take smaller bets in the financial markets in the regulated economy. Yet, as shown in panel A, initially the NFW follows a very similar path with $B = 0.5$ or without the controls. A significant difference builds in period 185 when outflows from country 1 hit the limit, and stay close to it afterwards, in the regulated economy. In the unregulated economy capital outflows keep increasing, reaching above 0.90 already in period 190. At the same time the effect on the real exchange rate is largely negligible until period 185. In the consequent periods, in the unregulated economy country 1 continues to accumulate wealth, its NFW position increases, and the real exchange rate appreciates more than in the regulated economy. Panel C shows that the impact of the regulation on the price of equity is substantial. The difference between the two paths becomes more substantial when country 1's NFW turns negative. This is so because country 1 is the major buyer of the domestic equity due to built-in preference for domestic goods.

We also want to point out the asymmetric effect of the terms of trade. For example, in period 190 the NFW/GDP of country 1 is 144.9% and for country 2 this indicator is -234.3%, a significantly larger magnitude. Most of this difference is explained by an exceptionally high terms of trade standing at 1.463. The effect is asymmetric because it dampens positive positions and amplifies negative positions.

Finally, observe that only the tail behavior of economic variables is distorted. As long as the NFW position does not deviate far form even distribution, namely zero, evolution of most macroeconomic indicators in the regulated and unregulated economies is nearly the same.

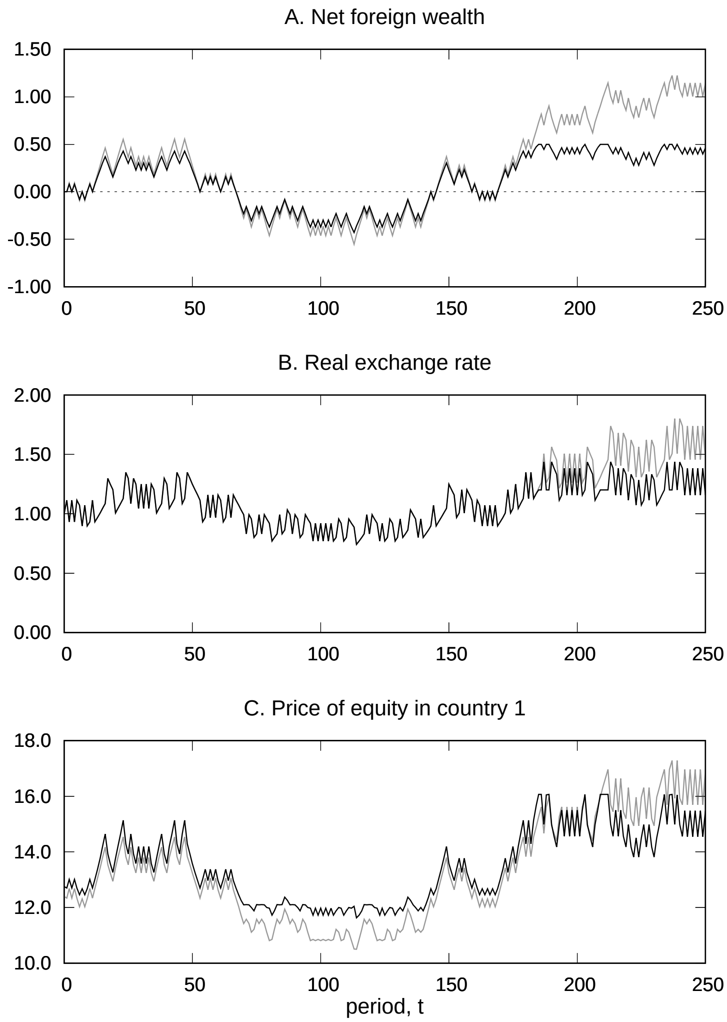

Figure 5: Selected simulated paths of the key macroeconomic variables for the unregulated economy (gray) and the economy with a net foreign wealth limit (black). Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

5.2 Transaction cost

Consider now taxing financial transactions at a fixed rate τ. The budget constraint of country i is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + \sum_j Q_j(z^t)[a_j^i(z^t) + \tau|a_j^i(z^t)|] = I^i(z^t) + T^i(z^t),$$

where $T^i(z^t)$ is the tax rebate received by country i. By the financial market clearing condition $\tau|a_j^1(z^t)| = \tau|a_j^2(z^t)|$; so, each country generates the same tax revenue. A country's budget constraint is unaffected because the tax collection is offset by the rebate; the good markets are not affected either. The effect on the economy comes about via distorted allocation and asset prices. Price of an Arrow security paying in state $z_{t+1} = j$ is determined from the following Euler equation:[17]

$$Q_j(z^t)(1 + \tau \cdot sign(a^i(z^t, j))) = \beta\pi^i(z^t, j|z^t)\frac{u'(c^i(z^{t+1}))}{u'(c^i(z^t))}. \tag{12}$$

So, if a country purchases Arrow security j it pushes the price up by a factor $1 + \tau$. The other country must sell the security, and, if wealth is evenly distributed, this restores the original security price. But for the security price to remain unchanged consumption in both countries must adjust. If wealth is not evenly distributed then we expect a non-trivial effect both on the security price and the consumption allocation.

Figure 6 shows the dynamics of the NFW, the real exchange rate, and the price of country 1's equity for the transaction tax level $\tau = 0.005$. For this magnitude of the tax the maximal value of country 1's wealth share is close to 0.5, just like under the NFW limit discussed above. The effect of the tax is that countries scale down their security purchases. As a result the path of the NFW of country 1 is a scaled down version of that under the unregulated markets as panel A demonstrates. More stable wealth position translates into a less volatile consumption process and, hence, a less volatile real exchange rate shown in panel B. The transaction tax also stabilizes the price of country 1's equity as can be seen from panel C.

The welfare surface for the economy with a transaction tax is very similar to the one with a NFW limit. So, it is not reported here. The main difference lies in the fact that transaction tax negatively affect welfare even if there is no disagreement. That is the measure's main disadvantage. A skeptic who believes that large disagreement is unlikely could point to this potential loss. At the same time, potential gains remain large but they are achieved at the cost of losing welfare even when there is no disagreement/speculation. Imposing a limit on NFW does not have this trade-off and hence if preferable in our view.

To conclude we would like to point out that the transaction cost should less favorable measure for the following reason. At the time of announcement

[17] Function $sign(x)$ is defined as $x/|x|$ if $x \neq 0$ and zero otherwise.

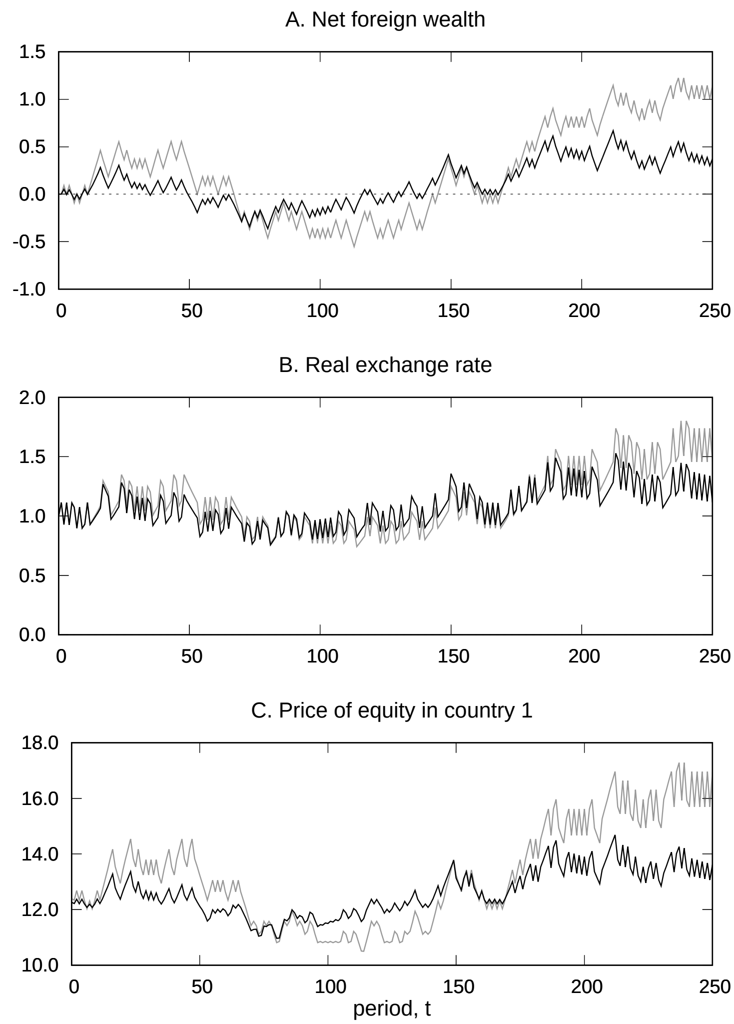

Figure 6: Selected simulated paths of the key macroeconomic variables for the unregulated economy (gray) and the economy with a transaction tax (black). Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

the transaction tax is going to have an immediate level effect on the asset markets: prices will be depressed because trading securities will become costlier. Announcement of a limit on current account will only have such effect if the country is close to exhausting this limit.

5.3 The effect of capital controls on asset markets

The proposed financial market restrictions impact the dynamics of the financial markets in unexpected way. To illustrate consider an example with symmetric beliefs as described in section 3 and three regimes: an unregulated economy, an economy with a NFW limit, and an economy with a transaction tax. We choose the transaction tax to make the last two setups have the same volatility of a country's NFW. In the case with low and high ES we set $\tau = 0.67\%$ and $\tau = 0.44\%$, respectively. We report the key financial market statistics in table 3. With either form of capital controls macroeconomic stability, as measured by volatility of consumption, CA and NFW, is improved significantly. Volatility of consumption decreases by a factor of 10, while that of NFW decreases at least by a factor of 3. In the case of low ES the capital controls also increases financial stability as measured by volatility of bond and equity prices. Asset price volatility decreases by at least a factor of 2. However, the same is not true if we assume a high value of ES. The case with high ES is closer to a one-good environment in which tighter regulation may be in conflict with financial stability, as observed in Blume et al (2015). To explain this phenomenon consider the case with the NFW limit. There are two driving forces of asset price volatility in the model: exogenous changes of countries' output and endogenous fluctuations in wealth distribution. Changes in wealth have a larger impact on asset prices in the case with low ES as the relative price of goods is more elastic. So, after a low output shock equity value in country 1 declines. Country 1's wealth and the price of good 1 decrease also, extending the decline of the equity value. So, capital controls, by restricting movements in wealth, stabilize equity prices. The effect of the NFW limit, a binding financial constraint that typically increases asset price volatility, is relatively unimportant. In the case with high ES the importance of the wealth effect and of the NFW limit are reversed. So, when the NFW limit is imposed asset price volatility increases.

The above analysis highlights that stabilization of capital flows may amplify fluctuations in domestic asset markets. When gross asset positions are substantial imposing capital controls may initially lead to increased volatility of capital flows. This should pose no problem if the financial system is sufficiently developed to withstand large asset price swings.

Finally, we would like to remark that the transaction tax is less effective at controlling CA fluctuations. In this case financial trade is costly but not impossible unlike under the NFW limit that halts trading when financial positions reach a given level. The transaction tax also depresses asset prices while the NFW limit increases them. The first is good for the case when the domestic equity market is "overvalued" because of capital inflows. The NFW limit is good when the domestic market declines due to capital outflows.

	$\sigma(\mathrm{NFA}^i)$	$\sigma(\mathrm{CA}^i)$	$\sigma(C^i)$	$\sigma(Q)$	$\sigma(q_b^i)$	$\sigma(q_e^i)$
A. Low elasticity: $\varepsilon = 0.8$						
Unregulated	1.275	0.130	0.049	0.593	0.146	0.130
NFW limit	0.331	0.066	0.004	0.192	0.044	0.065
Tobin's tax	0.337	0.064	0.012	0.211	0.045	0.056
B. High elasticity: $\varepsilon = 2.0$						
Unregulated	0.940	0.110	0.080	1.182	0.076	0.073
NFW limit	0.314	0.043	0.008	0.613	0.039	0.093
Tobin's tax	0.313	0.047	0.022	0.587	0.032	0.086

Table 3: Selected moments for the economies with and without capital controls. Because countries are symmetric we report moments only for one country. NFA and CA are normalized by a country's GDP, all other variables were transformed using the natural logarithm function. Asset prices are denominated in units of a domestic consumption basket.

6 The Case of Brazil

Brazil has been a major recipient of capital before the Global Financial Crisis (GFC) began. The country's net foreign liabilities increased more than threefold between 2003 and 2007, from 114.2 to 354.7 billion USD. The U.S. was and remains Brazil's major foreign investor. Its net investment nearly quadrupled, and its share reached 54.2% of the total net liabilities in 2007. Brazil experienced an unprecedented outflow of capital in 2008 and an even more significant inflow in 2009. The Central Bank of Brazil viewed the intense capital inflows as "excessive" with "the potential to threaten economic and financial stability".[18] Arguably, the largest worry of the Brazilian policymakers was the increased volatility of the value of its currency. Rapid appreciations experienced before the GFC and in 2010, see panel B in figure 7, were believed to hamper the country's competitiveness.

[18] See Tombini (2013).

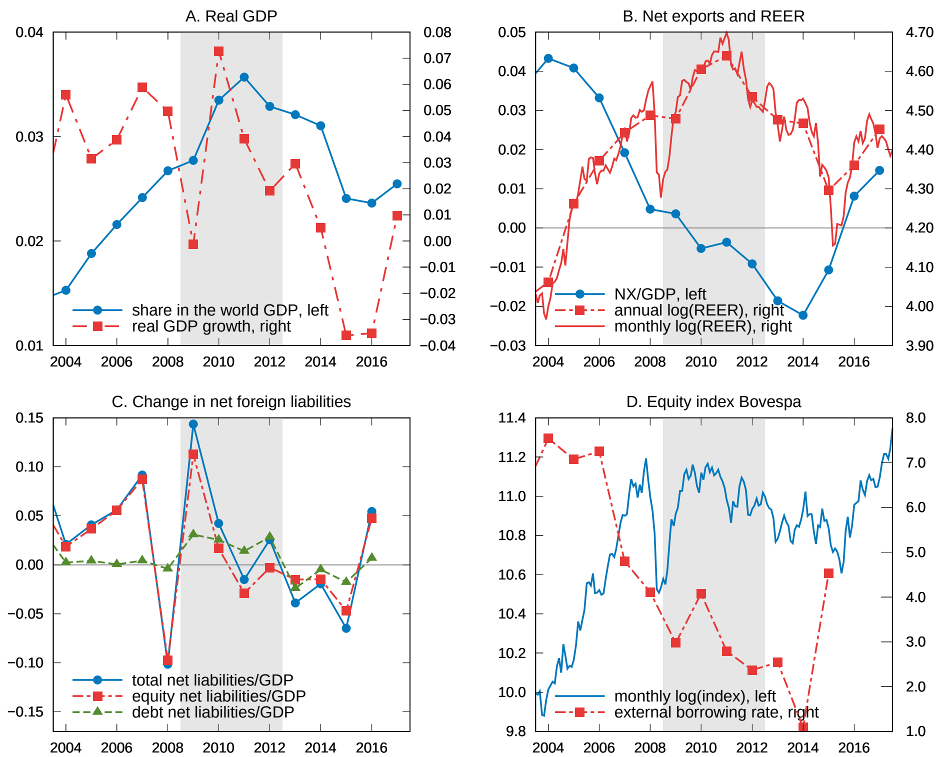

Figure 7: Brazil's economy during the period of active use of capital controls.

The model described in section 2 allows us to provide a new narrative for the dynamics of the Brazilian economy. We argue that the capital flows into Brazil could be driven by investor disagreement. Figure 8 shows the dynamics of the model calibrated to the Brazilian economy.[19] To avoid criticism that disagreement does not fade with time, the calibration assumes Bayesian learning from the realizations of Brazil's output. Disagreement arises because the investors start with different priors. The confidence of the foreign investors is equivalent to that built from an analysis of 100 years of data, while the investors in Brazil have infinite confidence in their beliefs. Because the Brazilian economy performed well before the GFC, the foreign investors are more optimistic than the domestic investors in the simulation. However, the level of disagreement is minimal at all times.[20]

[19] For the details on the calibration see section 6.1.

[20] An econometrician with a century worth of data would see the two beliefs as statisti-

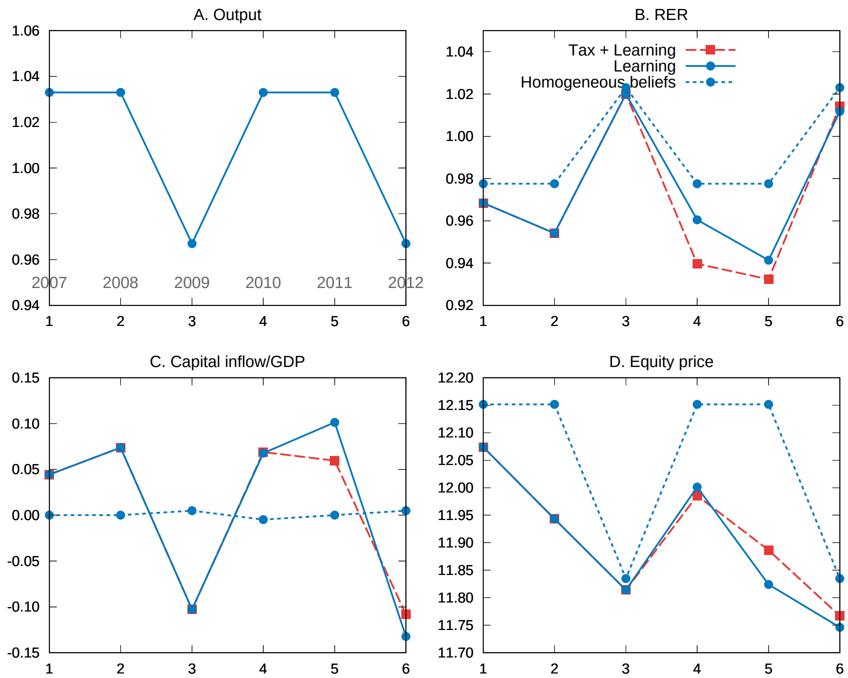

Figure 8: Simulation of the model calibrated to Brazil.

The dynamics of output mimics Brazil's GDP, the growth of which was below the long-run average of 2.4% in 2009 and 2012, see panel A in figure 7. In the benchmark model with homogeneous beliefs, capital flows would be minimal albeit non-zero. With learning, capital inflows initially increase and reach 6.7% of GDP in period 2. This inflow is part of the speculation strategy. The foreign investors are optimistic about Brazil, and purchase the portfolio that overpays, from the viewpoint of an investor with correct beliefs, when Brazil's economy performs. A substantial capital outflow of 10.2% of GDP, perfectly matching the data, realizes in period 3 when the economy's output declines. The outflow occurs because the optimistic foreign investors loose wealth after Brazil's economy declines.

Movements of the net wealth increase RER volatility: a country that grows its wealth sees its currency and equity appreciate. Brazil's net foreign wealth increases between period 2 and 3 creating an appreciation pressure, and decreases between periods 3 and 4 creating a depreciation pressure.

cally indistinguishable at the 99% or higher confidence level.

The volatility of (log) RER is higher in the economy with learning than homogeneous beliefs. The log-return of RER between periods 2 and 3 is 0.0667 and 0.0455 in the economy with learning and homogeneous beliefs, respectively. That is the RER increases by 46.6% more between period 2 and 3. An increase of Brazil's net foreign wealth in period 3 contributes to the RER increase.

Because there was a policy shift during the analyzed period, we also present the alternative simulation with a 3% financial transaction tax in the last three periods. Figure 8 shows the results denoted by the dashed line. The simulation with the transaction and learning coincides with the one without the tax before period 4, i.e., when there is no transaction tax. Expectedly, the tax imposed in periods 4-6 brings down the capital flows. The impact on the capital outflow in the last period is relatively mild, which is partly explained by the foreign investors expecting the tax to be temporary. The policy achieves one of its key goals: to contain appreciation. The net effect is a 2.2% and 1.0% depreciation in periods 4 and 5, respectively. The policy also seems to stimulate the domestic stock market. This effect can be explained by an increase in the state prices. With the transaction tax in place, it is costlier to transfer payoffs into the future.

6.1 Calibration for Brazil

The model has few parameters: volatility of output, foreign investor's prior, preference weight on the domestic good, risk aversion, and discount factor. Despite this relative simplicity, the model's predictions are grossly consistent with the data.

Table 4 reports the parametrization used for the simulation in figure 8 and the description follows. The output/productivity process is discretized into a two-equiprobable-state process. The output states are chosen to match the volatility of Brazilian real GDP detrended with a cubic polynomial. The output of the foreign economy is assumed to be fixed.

We assume that investors start with a Beta prior: $p_2^i = prob^i(s_2) \sim \mathcal{B}(n_1^i, n_2^i)$. Under this prior specification, the implied probability of the high output state is:

$$\hat{p}_2^i = n_2^i/(n_1^i + n_2^i).$$

We set $n_1^f + n_2^f = 100$ which determines the confidence level of the foreign investors. The ratio $n_1^f + n_2^f$ is set so that the simulated outflow in the period corresponding to 2008 is 10.2%. Because Beta is a conjugate prior the posterior is simple to compute:

$$posterior = \mathcal{B}(n_1^f + 1_{y_{t+1}^h = y_1}, n_2^f + 1_{y_{t+1}^h = y_2}),$$

where 1_X is an indicator function that returns 1 when the condition X is true and zero otherwise.

The home investors are assumed to be infinitely confident and correct: $n_1^h = n_2^h = \infty$. In practice, this amounts to setting $\hat{p}_2^h = 0.5$ at all times.

The prior of the foreign investors in the only degree of freedom in our model. We use it to match the capital outflow in Brazil in 2008.

The risk aversion parameter has been set to 1 (logarithmic preferences). The CES power and preference weight on domestically-produced goods were set at -0.25 and 0.90, respectively. Together they imply that the elasticity of substitution between goods is 0.80 and that the net exports to GDP ratio would be 0.85 when Brazil's GDP equals its expected value.

The discount factor was chosen so that the real interest rate is 4% when Brazil's GDP is fixed at its expected value.

Parameter	Moment	Value
$\sigma(e)$	Volatility of detrended output	0.033
$\mathcal{B}(n_1^f, n_2^f)$	Prior of the foreign investors	$\mathcal{B}(49.5, 50.5)$
$\mathcal{B}(n_1^h, n_2^h)$	Prior of the domestic investors	$\mathcal{B}(\infty, \infty)$
ρ	CES power parameter	$-.250$
s	CES weight on dom.-produced goods	0.910
γ	CRRA risk-aversion parameter	1.000
β	Discount factor	0.962

Table 4: Parameters of the model calibrated to Brazil's economy.

7 Conclusions

We study an international portfolio choice model with heterogeneous beliefs. Belief diversity generates speculative cross-border capital flows that are much larger than flows generated by the hedging/insurance motives. Capital controls improve welfare because they limit movement of the relative wealth positions and, hence, consumption. Using numerical simulations, we show that the capital controls that restrict tails of the capital flows and exchange rates are most desirable. That is the controls that activate only during substantial inflows or outflows are preferred to those that are constantly active.

This work opens up a possibility to study the recent episodes of speculative capital flows quantitatively. Are the capital flow controls expected to work as intended if a policy is enacted after massive flows taken place? How effective are unilateral measures and can they fire back? The setting with heterogeneous beliefs also offers numerous advantages – from resolving the long-standing puzzles in the international finance like consumption-exchange rate disconnect to offering a plausible explanation of the phenomena such as excess return volatility and return predictability. Building upon this model allows matching the dynamics of observed capital flows, and heterogeneous beliefs can be easily included into existing models of macro-prudential regulation.

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A Efficient allocations

Here we derive a condition that must be satisfied by any CE allocations as long as trade in goods is frictionless. The derivations below assume that the consumption aggregator is CRS. The first order conditions for (c_1^i, c_2^i) are:

$$\beta\pi^i(z^t)u'(c^i(z^t))g_1(c_1^i(z^t), c_2^i(z^t)) = \lambda p_1(z^t),$$
$$\beta\pi^i(z^t)u'(c^i(z^t))g_2(c_1^i(z^t), c_2^i(z^t)) = \lambda p_2(z^t).$$

Multiply the two equations by c_1^i and c_2^i respectively and add to get:

$$u'(c^1)c^1 = \lambda(p_1 c_1^1 + p_2 c_2^1) = \lambda p^1 c^1.$$

The above implies that $g_1(c_1^i, c_2^i) = p_1/p^i$ and:

$$\frac{g_1(c_1^2, c_2^2)}{g_1(c_1^1, c_2^1)} = \frac{p^1}{p^2} = Q. \tag{13}$$

B RER is bounded

In this appendix we establish that the real exchange rate RER is bounded in any complete markets competitive equilibrium.

Let $h^i(x,y) = g_1^i(x,y)/g_2^i(x,y)$. Because the trade in goods is frictionless the following relation must hold:

$$h^1(c_1^1, c_2^1) = h^2(e_1 - c_1^1, e_2 - c_2^1) = p_1/p_2. \tag{14}$$

In the case of the CES good aggregator $h^1(x,y) = \frac{s}{1-s}(x/y)^{\rho-1}, h^2(x,y) = \frac{1-s}{s}(x/y)^{\rho-1}$. The equation (14) can be solved for c_1^1 in terms of c_2^1:

$$\frac{c_1^1}{c_2^1} = \frac{e_1}{e_2}\frac{1}{\chi(1 - c_2^1/e_2) + c_2^1}, \ \textit{where } \chi \equiv \left(\frac{s}{1-s}\right)^{-2\varepsilon} \leqslant 1.$$

The terms of trade, then, can be expressed as a function of c_2^1:

$$q = \left[(e_2/(\sqrt{\chi}e_1))(\chi(1 - c_2^1/e_2) + c_2^1/e_2)\right]^{1/\varepsilon}. \tag{15}$$

Because $\chi \leqslant 1$ the terms of trade is an increasing monotone function of c_2^1. Intuitively, when consumption in country 1 increases, this country's terms of trade improve. The bounds on consumption imposed by resource feasibility, $c_2^1 \in [0, e_2]$, imply that the terms of trade is constrained to $[x^{1/(2\varepsilon)}, x^{-1/(2\varepsilon)}]$.

Finally, the real exchange rate is a continuous function of the terms of trade:

$$Q = \left[\frac{q^{1-\varepsilon} + \sqrt{x}}{\sqrt{x}q^{1-\varepsilon} + 1}\right]^{\frac{1}{1-\varepsilon}}, \ \textit{where } q = p_1/p_2. \tag{16}$$

Hence, Q is bounded because q is.

C Dampening

The Pareto optimality condition is:

$$ln(u'(c^1)/u'(c^2)) - \ln(Q) = exogenous$$

Suppose that country 1's consumption declines or, equivalently, that its wealth declines. Then the analysis above implies that the terms of trade of this country declines. Intuitively, the terms of trade worsen because country 1's wealth declines and it values good 1 more than the rest of the world. That is:

$$cor(ln(u'(c^1)/u'(c^2), -\ln(Q)) > 0.$$

That is, the adjustment in the real exchange rate smooth out the decline in consumption when a country loses wealth. When a country loses wealth value of the good produced by this country declines. The country shifts its consumption away from foreign goods. As a result, its aggregate consumption does not decline as much as it would had the relative good prices remained constant.

D Welfare in financial autarky

Consider the financial autarky. For the optimization problem of each country is static we drop the time subscript in this subsection. Optimal trade in goods between countries solves the following system of equilibrium equations for any pair $(e_1, e_2) \gg 0$:

$$g_1^i(c_1^i, c_2^i)/g_2^i(c_1^i, c_2^i) = q, \quad i = 1, 2, \tag{17a}$$

$$c_j^1 + c_j^2 = e_j, \quad j = 1, 2, \tag{17b}$$

$$q(c_1^1 - e_1) + c_2^1 = 0. \tag{17c}$$

Given that g^1, g^2 satisfy the Inada condition it is easy to show that c_1, c_2 is an interior solution. The optimal allocation is a continuous function of (e_1, e_2). Because the latter is bounded the consumption of each good must also be bounded, most importantly, below.

E Welfare in unregulated economy

This section complements the discussion in section 3 by describing how welfare changes with countries' beliefs. Because endowments are symmetric under the true dgp we report only welfare of country 1 in figure 9. First, welfare of country 1 is high when its beliefs are more accurate. In particular, it is also higher than in the homogeneous beliefs case (point C), although concavity of the period utility function limits the gain at 1.6%. But belief accuracy is not the only force. For example, at $(p^1, p^2) = (0.50, 0.45)$ country 1's information advantage is maximized, but its welfare is not. Under belief assignment $(p^1, p^2) = (0.50, 0.55)$ country 1's welfare is higher despite the informational advantage being the same. The reason is that under the latter belief assignment the world is biased towards thinking that country 1's expected output is higher. This affects the price system in favor of country 1 and allows it to achieve higher consumption. Country 1's welfare is still higher at $(p^1, p^2) = (0.525, 0.55)$ denoted by point A_1 where some of the informational advantage is given up and expectation are biased more in the country's favor. Country 1's welfare is also very high at $(p^1, p^2) = (0.48, 0.45)$ denoted by point A_2, but not as high as at A_1 because these beliefs are biased against country 1.

Country 1 welfare can lose as much as 4.5% relative to the benchmark with correct beliefs. The lowest welfare is achieved at $(p^1, p^2) = (0.45, 0.535)$ denoted by point B_1 where country 1 has less accurate beliefs, disagreement is nearly maximized, and expectations are stacked against it. Its welfare is very also at $(p^1, p^2) = (0.55, 0.46)$ denoted by point B_2, but not as low as at B_1 because these beliefs are slightly biased in favor of country 1.

Despite the saddle-shaped welfare function of each individual country, the total welfare in the world is concave surface with a maximum at point C where both countries hold correct beliefs. Welfare does not decline more than 0.3% if countries hold the same beliefs. But the loss increases to 4.3% at the points where disagreement is maximal: $(0.45, 0.55)$ or $(0.55, 0.45)$.

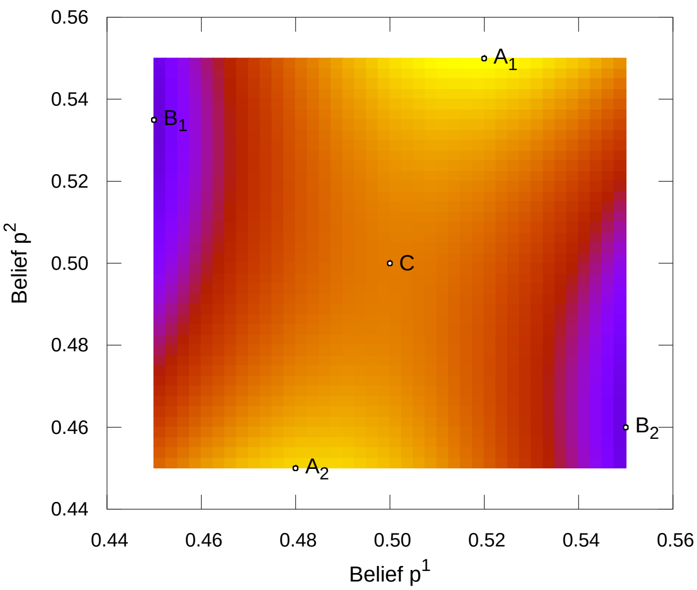

Figure 9: Welfare in the unregulated economy of section 3 for a range of possible belief assignments.

F Dynamics with equally incorrect beliefs

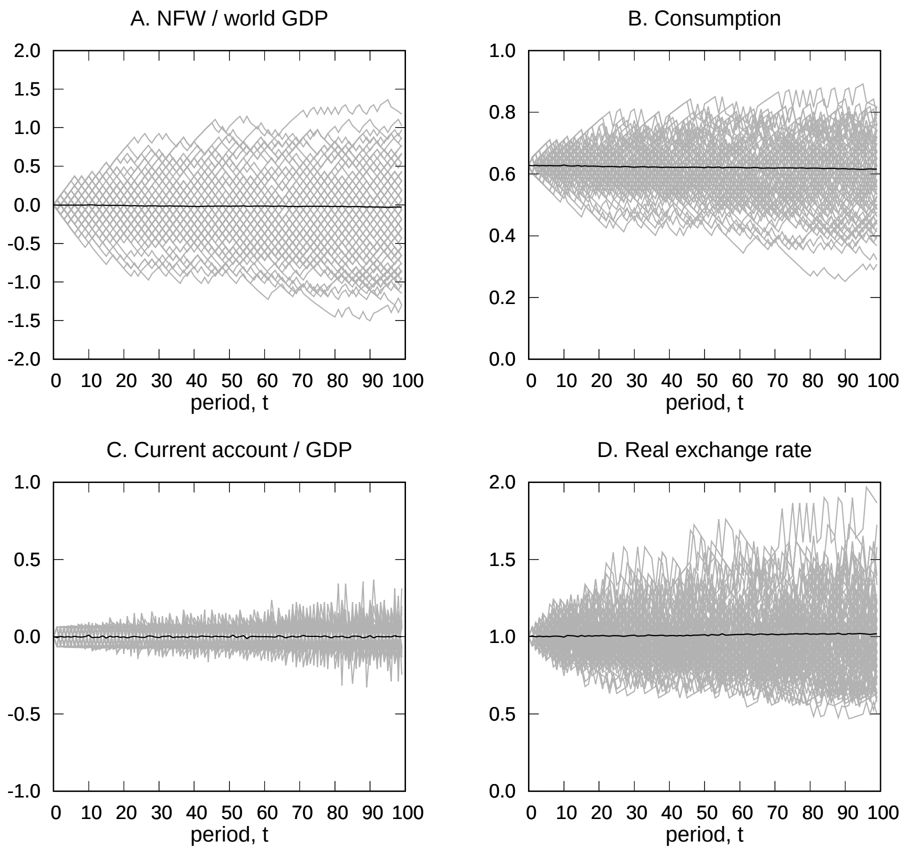

Figure 10: Dynamics of country 1. Countries have wrong but equally-inaccurate beliefs. Grey lines represent 100 random paths, black lines represent the averages across paths. The variation of the averages is due to the sampling error.

Figure 10 plots key macroeconomic indicators for the economy in which both countries are optimistic about their own prospects. Endowments are specified as in section 3 and beliefs are:

$$\pi^1 = \begin{bmatrix} 0.525 \\ 0.475 \end{bmatrix}, \pi^2 = \begin{bmatrix} 0.475 \\ 0.525 \end{bmatrix}, \quad \pi^0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

G The cost of business cycles

The cost of business cycles is measured by λ that solves the following equation:[21]

$$U^i(\lambda c^i) = U^i(E_0(c^i)). \tag{18}$$

[21] See Lucas (2003).