Trading on Sunspots

By Boyan Jovanovic and Viktor Tsyrennikov *

In a model with multiple Pareto-ranked equilibria, we show that the set of equilibria shrinks if we allow trade in assets that pay based on the realization of a sunspot acting as an equilibrium-selection device. When the probability of a low-output outcome is high, the desire to insure against it leads the poor to promise large transfers to the rich in the high-output state. The rich then lose the incentive to exert the effort needed to sustain the high output. Thus the opening of financial markets may destroy the high equilibrium.

We study how asset markets relate to coordination failures. We start with a model in which in the absence of asset markets multiple equilibria arise. A sunspot, an extrinsic public signal, correlates agents' actions and selects the equilibrium outcome. We then add trade in sunspot-contingent assets and find that opening financial markets may limit the set of equilibria and in particular may destroy good equilibria, thus having a perverse effect in the economy. In other words, the high equilibrium may cease to exist if one allows for trade in sunspot contingent financial assets.

The model has two types of agents, endowment-rich and endowment-poor, producing output and engaging in financial trade. The coordination game has two equilibria – "low" and "high" – that are selected by the sunspot state. Aggregate output, consumption, and employment are low in the low equilibrium. Both types face aggregate risk, and each type is better off in the high equilibrium. The poor buy insurance against the low outcome, and the portfolio of the rich then pays when the outcome is high. The higher the chance of the low outcome, the larger the payoff in the high state. But as their utility of consumption is concave, the rich will not want to exert effort when their financial income is high enough. The high outcome then fails to be an equilibrium and the financial market has no trade.

The size of the financial payoff by the poor to the rich determines the upper bound on the probability of the low equilibrium. It depends on preferences, production, and endowments. For example, the higher the inequality of initial endowments, the higher are the financial trades and the larger is the set of parameters for which the high equilibrium fails.

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The shaded areas of figure 1 shows the combinations of the low-sunspot probability, π^l, and relative risk aversion, γ, for which both low equilibrium (L) and high equilibrium (H) exist. Panel A corresponds to a two period and panel B to an infinite horizon models, respectively. As γ rises, agents want more insurance and take larger financial positions. In turn, large financial payoffs destroy the incentives to exert effort, and the high outcome ceases to exist. Denoted by the solid line in the figure, the upper bound on π^l falls.

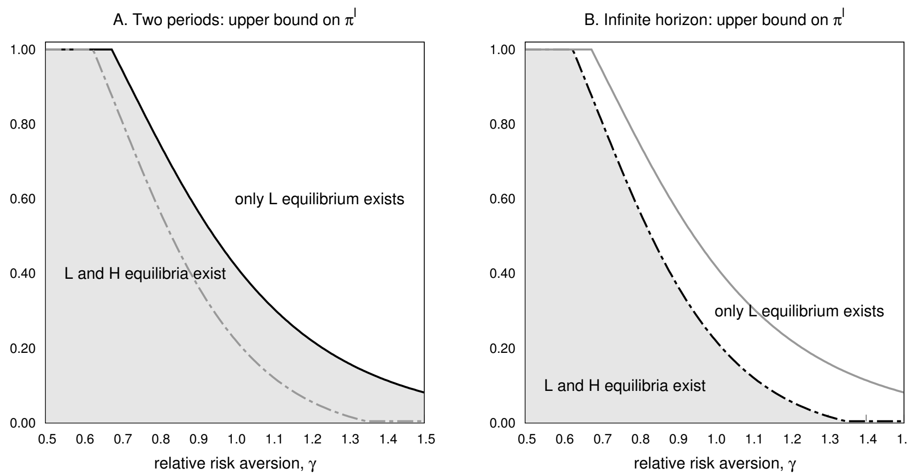


Figure 1. : The upper bound on π^l that is consistent with equilibrium when financial markets are open for a range of CRRA γ. The solid and dash-dot lines correspond to the static and infinite horizons models, respectively.
Parameters are reported in Table A1

This result is in the spirit of Lucas (1976). He argued that one could not use estimated responses to monetary policy to evaluate changes in the stochastic process governing money injections because agents' responses depend on their expectations. If the distribution of money injections shifts, agents' responses to a given money injection change. A parallel arises here: If the distribution of the sunspot shifts, agents' responses to a given sunspot realization may change. A higher π^l induces them to raise the amount of insurance for the low sunspot state and to pay correspondingly more in the high sunspot state and thereby eliminate the recipients' incentives to work in the high state. In panel B, the sunspot has no effect on economic activity, which remains low, in the region between the solid and dashed lines.

When the sunspot selects an equilibrium, aggregate output is uniquely determined. We show that markets for equity and a safe asset suffice to implement sunspot-contingent trades. Because the rich receive transfers from the poor in the high state, consumption and wealth inequality are procyclical.[1]

The paper's main result, presented in Sections 2 and 3, is about the effects of insurance (trading Arrow securities on sunspots) against the bad equilibrium on output. In a pure sunspot example, in an infinitely repeated sunspot economy, and in an example with sunspots and exogenous shocks, we show that the possibility of insurance can lead to a breakdown of the good equilibrium and hence might decrease welfare. If the equilibrium set stays intact finance raises welfare. But if the high equilibrium is destroyed, welfare falls.

Section III then shows that the argument extends to an infinite horizon economy. And Section IV shows that the essence of the argument applies in a setting where equilibrium is unique but in which there are real shocks; agents then trade real-shock-contingent assets and the wealth effect of rich agents reduces their effort when the shock is high. Equilibrium still exists, but in the high state aggregate output is lower than it would be in the absence of financial trade.

The related literature covers several broad topics: Contracting in the presence of hidden borrowing and lending, sunspot-contingent commodity prices, bank runs and speculative attacks, and sovereign debt crises.

Contracts and "outside" assets.—Closest to our model and results, contracting failures have been linked to "outside" asset trading that is not a part of the contract. Allen (1985), Cole and Kocherlakota (2001), and Bisin and Guaitoli (2004) show that the possibility of outside borrowing and lending and, more generally, non-exclusive contracting weakens the power of contracts to discipline agents and changes equilibrium actions. Similarly, adding asset trading to the model of Golosov and Menzio (2020) would lead agents to purchase securities paying in the sunspot states in which they are fired, and that would weaken their incentives to work. Our results are in line with these implications – it is the high-output outcome that contracting may destroy.

Sunspot-contingent output and commodity prices.—Peck and Shell (1991) and Forges and Peck (1995) assume that commodity prices depend on sunspots under complete financial markets. In Bhattacharya, Guzman and Shell (1999) some agents are restricted from trading sunspot-contingent assets or commodities. Work on coordination failures as causing business cycles dates back to Cooper and John (1988) and Benhabib and Farmer (1999) argued is a worthwhile goal of sunspot models.

Bank runs, speculative attacks.—Multi-investor situations often involve multiple equilibria. A bank run will cause a drop in future output when high-return long-term investments are not funded. Cooper and Ross (1998) derive the largest

[1]For most households in the U.S., the main component of wealth is housing, the value of which is pro-cyclical according to Piazzesi and Schneider (2016). Batty *et al.* (2019) report that the wealth share of the top 1% in the U.S. has been procyclical since 1989.

probability of a run in a sequential-service bank contract above which depositors will not want to enter into sequential-service contracts. Further, in Peck and Shell (2003), banks offer less restrictive financial contracts to prevent bank runs. Second, Freeman (1988) and Bental, Eckstein and Peled (1991) study bank contracts that allow dependence of a deposit contract on the sunspot, and Goldstein and Pauzner (2005) derive a unique probability of a bank-run using a global game argument. Another strand studies how government intervention affects the equilibrium – Keister (2016).

Sovereign debt.—In our model financial markets may reduce the number of equilibria. By contrast, in sovereign debt models like Calvo (1988) and Cole and Kehoe (2000), they may do the opposite: An equilibrium can exist in which interest rates are low and in which government debt sells at a high price, and then the government does not default. But another equilibrium may exist in which the price of debt is low, the interest rate is high, and the government defaults.

Plan of paper.—Section I begins with a model without financial markets and then shows how opening financial markets restricts the equilibrium set. It then describes asset pricing with disaster risk. Section ii specializes to log utility. Section III generalizes to an infinite horizon. Section IV studies the real version of the model in which equilibrium is unique. Section V concludes the paper, and the longer proofs are in the Appendix.

I. The model

Consider a production economy with individuals with preferences described by a utility function that depends on consumption $c \geqslant 0$ and effort x.

$$U(c) - \kappa x, \tag{1}$$

where κ is the utility cost of effort, and where U is increasing and strictly concave.

Production.—Agents are of two types $i \in \{1, 2\}$, and their population fractions are f_i. A type i agents output is

$$y(x, \bar{x}) = (\alpha + \bar{x})x_i.$$

where x_i is effort of agents of type i, and where $\bar{x}$ is aggregate effort[2]

$$\bar{x} = f_1 x_1 + f_2 x_2.$$

Endowments and consumption.—Without financial markets, type-i agent's consumption is $z_i + (\alpha + \bar{x})x_i$, where z_i is the agent's endowment, with $0 < z_1 < z_2$.

[2]The term $y(1, \bar{x})$ could be interpreted as the wage per unit of x offered by competitive firms if aggregate effort is $\bar{x}$. Consistent with evidence in Basu and Fernald (2001), productivity of effort, $\alpha + \bar{x}$, will be pro-cyclical.

The aggregate endowment is

$$\bar{z} = f_1 z_1 + f_2 z_2.$$

We focus on the coordinated equilibria with $\bar{x} = 0$ or $\bar{x} = 1$. An agent's output level then is

$$Y = \begin{cases} 0 & \textit{if low output} \\ 1+\alpha & \textit{if high output} \end{cases}.$$

Hence, consumption of type i is

$$C = \begin{cases} z_i & \textit{if low output} \\ z_i + 1 + \alpha & \textit{if high output} \end{cases}.$$

The two types' consumption shares in the low-output relative to the high-output state are ordered as follows:

$$\frac{z_1}{z_1 + 1 + \alpha} < \frac{z_2}{z_2 + 1 + \alpha}.$$

With CRRA preferences, the low-z or "poor" agent will have a relative preference for consumption in the low output state. Since effort costs κ are the same for all, the poor agent will have the greater incentive to deviate from the low equilibrium, and the rich agent will have the greater incentive to deviate from the high equilibrium.

When financial markets allow sunspot-contingent trades, the rich will trade consumption in the low state in exchange for consumption in the high state because of the relative consumption result and CRRA preferences.[3] This will weaken the incentive for the rich agent to work in the high output state, and will weaken the incentive for the poor agent to work in the low output state. However, the stronger effect here is with respect to the rich agent in the high output state, and if z_2 is much larger than z_1 the high equilibrium may disappear.

In section III we analyze an infinitely repeated version of the game in which the introduction of finance allows agent to smooth consumption via the asset market. We find that the high equilibrium can be more or less likely to survive depending on parameters. On the one hand, the rich can smooth some of their windfall gains thereby raising their high-equilibrium utility but on the other, they also can smooth the income loss entailed in their deviating from that equilibrium.

A. *Equilibria with no financial markets*

We start with a setting in which there are no financial markets. Until section IV the model has no intrinsic shocks.

Sunspots.—A sunspot is an extrinsic random variable s that assumes two values:

[3] In fact, $U''' > 0$ is enough for our results to be true, but we will rely on the CRRA form to derive analytical results.

l and h. Agents can coordinate their actions on the sunspot state $s \in \{l, h\}$. Additionally, we restrict attention to symmetric pure strategy equilibria in which all agents of one type work the same amount so that an agent's x depends only on z and on the sunspot realization. Effort x_i^s is then the only action.

Equilibrium with no assets.—An equilibrium is a function x_i^s such that for all $(i, s) \in \{1, 2\} \times \{l, h\}$,

$$x_i^s \in \arg\max_{x \in \{0,1\}} \left\{ U\big(z + y(x, \bar{x}^s)\big) - \kappa x \right\} \tag{2}$$

where

$$\bar{x}^s = f_1 x_1^s + f_2 x_2^s. \tag{3}$$

Equilibrium "L".—In the first type of equilibrium, denoted by "L" or "low", agents do not work: $x_z^s = \bar{x}^s = 0$. For this equilibrium to exist, both agent types must weakly prefer not to work:

$$U(z_1) \geqslant U(z_1 + \alpha) - \kappa, \tag{4a}$$

$$U(z_2) \geqslant U(z_2 + \alpha) - \kappa. \tag{4b}$$

That is, if $\bar{x}$ is zero, the reward to effort is just α, and each type should prefer not to work. Because U is concave it is sufficient that the *poor* are *not* willing to work:

$$U(z_1 + \alpha) - U(z_1) \leqslant \kappa. \tag{5}$$

Equilibrium "H".—In the second type of equilibrium, denoted by "H" or "high", every agent works: $x_z^s = \bar{x}^s = 1$. For this equilibrium to exist, both agent types must weakly prefer to work:

$$U(z_1 + \alpha + 1) - \kappa \geqslant U(z_1), \tag{6a}$$

$$U(z_2 + \alpha + 1) - \kappa \geqslant U(z_2). \tag{6b}$$

Because U is concave, it is sufficient that the *rich are* willing to work:

$$U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa. \tag{7}$$

In equilibria H and L, all agents take the same action – either every agent works or no agent does.[4]

[4]In this type of a game, the number of equilibria is generically odd. This means that when the L and H equilibria both exist, there will also be a third "mixing" equilibrium, which we denote by "M." Only the endowment-poor work and $\bar{x} = f_1$ in the M equilibrium. For this equilibrium to exist, the poor and

The set of parameters under which both H and L exist, *i.e.*, (5) and (7) hold, is $\mathcal{P}$:

DEFINITION 1: *Let* $\mathcal{P} = \{(z_1, z_2, \kappa) : U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa \geqslant U(z_1 + \alpha) - U(z_1)\}$.

The set of parameters for which both L and H exist, *i.e.*, the set $\mathcal{P}$ is always non-empty. To see this fix $\alpha > 0$. Then for any $z > 0$ consider the following parameters: $z_1 = z, z_2 = z + \epsilon, \kappa = U(z + \alpha + 0.5) - U(z)$ for some small $\epsilon > 0$. Since the point is strictly inside $\mathcal{P}$, the latter must have a non-empty interior.

For a given κ, the above conditions translate into simple bounds on the endowments. Intuitively, endowment z_1 must not be too low as then type-1 individuals would always work and equilibrium L would not exist. Endowment z_2 must not be too high as then type-2 individuals would never work and equilibrium H would not exist.

LEMMA 1: *For a given* $\kappa \in (0, U(\alpha) - U(0)]$, *there exist unique* $z_{max} > z_{min} > 0$ *such that* $\mathcal{P} = \{z_1, z_2, \kappa : z_{min} \leqslant z_1 \leqslant z_2 \leqslant z_{max}\}$.

PROOF:
The boundaries z_{min}, z_{max} solve the following equations:

$$U(z_{min} + \alpha) - \kappa = U(z_{min}),$$
$$U(z_{max} + \alpha + 1) - \kappa = U(z_{max}).$$

The above equations always have a solution if $\kappa \leqslant U(\alpha) - U(0)$. The solution is unique because U is strictly increasing and concave.

Figure 2 illustrates $\mathcal{P}$ and the sets where only one or none of the equilibria exist.

If agents coordinate their actions on the sunspot state, $s \in \{l, h\}$ determines aggregate effort $\bar{x}$ and aggregate output

$$Y^s \equiv \begin{cases} \alpha + 1 & \text{if } s = h \\ 0 & \text{if } s = l \end{cases} . \tag{8}$$

We refer to the equilibria $E \in \{L, H\}$ and aggregate output interchangeably so that $E = L \Leftrightarrow Y = 0$ and $E = L \Leftrightarrow Y = \alpha + 1$.

rich agents must weakly prefer to work and not work, respectively:

$$U(z_1 + \alpha + f_1) - \kappa \geqslant U(z_1),$$
$$U(z_2 + \alpha + f_1) - \kappa < U(z_2).$$

We assume that equilibrium M is never chosen, as we focus on the probability of the low output outcome – a "disaster."

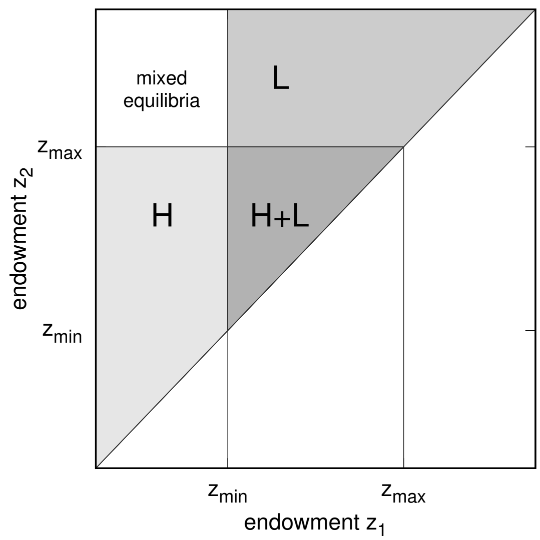


Figure 2. : Equilibrium map under the financial autarky. This figure corresponds to any set of parameters for which $\mathcal{P}$ is non-empty.

B. *Equilibria with financial markets*

We now describe how opening financial markets affects the equilibrium set.

Financial markets.—There are markets for two Arrow securities. Each security is in zero net supply, and security s pays a unit of consumption if sunspot state s realizes tomorrow and zero otherwise. Let Q^s be the price of the security paying in state s, and let n_i^s denote agent i's investment in that security. If all agents of a given type choose the same portfolio, as we assume, market clearing requires that for each security s,

$$f_1 n_1^s + f_2 n_2^s = 0. \tag{9}$$

Budget constraint.—Because agents have no income in the first period their purchases of securities must have zero net value. That is, the budget constraint for agent $i \in \{1, 2\}$ is

$$Q^l n_i^l + Q^h n_i^h = 0. \tag{10}$$

In financial autarky, *i.e.*, when financial markets are closed, $n_i^s = 0, \forall i, s$.

Timing.—Figure 3 describes the timing of events.

Equilibrium with financial markets.—It is a list of functions $Q : \{l, h\} \to R_{++}$, and $(x_i^s, n_i^s) : \{l, h\} \to \{0, 1\} \times R$ for $i = 1, 2$ such that

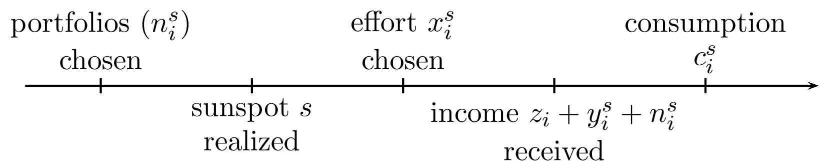


Figure 3. : Timing of events

1) Actions (x_i, n_i) are optimal, *i.e.*, they solve

$$\max_{n_i^l, n_i^h} \sum_s \pi^s \cdot \max_{x_i^s} \left[U(z_i + y(x_i^s, \bar{x}^s) + n_i^s) - \kappa x_i^s \right]$$

subject to the budget constraint (10). There is no commitment to x_i^s; they are optimal *ex post* and so they appear inside the summation sign.

2) Financial and good markets clear at each s

$$f_1 n_1^s + f_2 n_2^s = 0,$$
$$f_1 c_1^s + f_2 c_2^s = \bar{z} + Y^s.$$

In the model with financial markets, we assume that equilibrium L occurs if $\bar{x}^l = 0$, and equilibrium H occurs if $\bar{x}^h = 1$. We now examine the set of disaster probabilities π^l for which equilibria L and H both survive. We call such π^l values "admissible."

Optimality conditions

The first-order necessary conditions imply that for each individual the marginal rate of substitution between consumption in states h and l equals the relative price:

$$q \equiv \frac{Q^h}{Q^l} = \frac{\pi^h}{\pi^l} \cdot \frac{U'(z_i + \alpha + 1 + n_i^h)}{U'(z_i + n_i^l)}, \quad i \in \{1, 2\}. \tag{11}$$

The above implies that the marginal rate of substitution is the same across individuals: This is a standard risk-sharing result that obtains here because markets are complete.

The security prices Q^l and Q^h are also referred to as state prices, and q is the relative state price. Only q matters and the following lemma is important for the equilibrium characterization.

LEMMA 2: *If U is homothetic, then the relative state price is*

$$q = \frac{\pi^h}{\pi^l} \cdot \frac{U'(\bar{z} + Y^h)}{U'(\bar{z} + Y^l)}. \tag{12}$$

PROOF:

The first order optimality condition implies that

$$\frac{U'(c_1^h)}{U'(c_2^h)} = \frac{U'(c_1^l)}{U'(c_2^l)}.$$

Denoting agent i's share of consumption by η_i^s, preference homotheticity implies

$$\frac{U'(c_1^s)}{U'(c_2^s)} = \frac{U'(c_1^s/(\bar{z} + Y^s))}{U'(c_2^s/(\bar{z} + Y^s))} = \frac{U'(\eta_1^s)}{U'(1 - \eta_1^s)}.$$

Since $U'' < 0$, the above ratio of marginal utilities is strictly decreasing in η_1^s. Because the ratio of the marginal utilities must be the same across states, we get

$$\eta_1^l = \eta_1^h \equiv \eta_1. \tag{13}$$

After invoking preference homotheticity again, we obtain the desired result

$$q = \frac{\pi^h}{\pi^l} \frac{U'(c_1^h)}{U'(c_1^l)} = \frac{\pi^h}{\pi^l} \frac{U'(\eta_1(\bar{z} + Y^h))}{U'(\eta_1(\bar{z} + Y^l))} = \frac{\pi^h}{\pi^l} \frac{U'(\bar{z} + Y^h)}{U'(\bar{z} + Y^l)}.$$

For given $\alpha, \bar{z}$, Lemma 2 shows that the relative likelihood of states h and l is its single driving force. Intuitively, the lower the probability of a state the lower is demand for the security paying in this state. The following lemma states that $q = Q^h/Q^l$ decreases in π^l.

LEMMA 3: *q is decreasing in π^l.*

PROOF:

By direct differentiation.

Remark for the zero-trade case.—This arises when $\pi^l > \bar{\pi}^l$. In that case the high equilibrium disappears, $Y^h = Y^l = 0$. Financial markets remain open but there is no trade and, because consumption is the same in states l and h, the relative security price is

$$q = \frac{\pi^h}{\pi^l}, \quad \forall \pi^l > \bar{\pi}^l.$$

Consumption risk-sharing

The previous section asserts that each agent i consumes a constant fraction η_i of aggregate income, *i.e.*, the sum of aggregate endowment and production output:

$$c_i^s = \eta_i(\bar{z} + Y^s). \tag{14}$$

This relationship implies that $c_i^l < c_i^h, i = 1, 2$ because $Y^l < Y^h$. We can also show that the rich will consume a larger fraction of total income. To this end, using the budget constraint (10) we get

$$\eta_i = \frac{z_i + Y^l + q(z_i + Y^h)}{\bar{z} + Y^l + q(\bar{z} + Y^h)} \in [0, 1].$$

Because $z_2 \geqslant z_1$, the rich consume a larger share of the total good supply, *i.e.*, $\eta_2 > \eta_1$ for any $q > 0$, as lemma 4 states.

LEMMA 4: *With (or without) financial markets open,* $c_i^l \leqslant c_i^h, \forall i$ *and* $c_1^s \leqslant c_2^s, \forall s$.

Letting $\Delta_z \equiv z_2 - z_1$, equation (14) and budget constraint $c_i^s = z_i + Y^s + n_i^s$ imply

$$n_1^l = \eta_1[\bar{z} + Y^l] - z_1 - Y^l \leqslant \bar{z} - z_1 = f_2\Delta_z, \tag{15a}$$

$$n_2^l = -(f_1/f_2)n_1^l \geqslant -f_1\Delta_z > -\Delta_z. \tag{15b}$$

An important special case arises when $\pi^l = 1$, *i.e.*, equilibrium H occurs with probability 0. In this limit case

$$\eta_i\Big|_{\pi^l=1} = \frac{z_i + Y^l}{\bar{z} + Y^l}. \tag{16}$$

Optimal portfolios

To understand portfolio decisions consider the case when financial markets are closed. While a low-endowment type-1 individual has lower utility in every state his relative marginal value of consumption in the low output state is higher

$$\frac{U'(z_1)}{U'(z_2)} > \frac{U'(z_1 + Y^h)}{U'(z_2 + Y^h)}.$$

A sufficient condition for the above to hold is a decreasing absolute risk aversion that, in turn, is true if $U''' > 0$. So, one should expect the low-endowment type z_1 to buy securities that pay in state $s = l$, $n_1^l \geqslant 0$, and sell securities that pay in state $s = h$ ($n_1^h \leqslant 0$) as conjectured above. This intuition will be used to derive sufficient conditions for existence of equilibria H and L.

Equilibrium H is vulnerable because the wealthier type-2 agent's incentives to exert effort could be weakened by financial trade. Type-2 agents insure the poorer type-1 agents as we argue above. Their incentives to exert effort fall because of positive financial income in equilibrium H. The formal argument follows below.

Homotheticity of preferences implies (see proof of lemma 2) that

$$\frac{z_i + Y^h + n_i^h}{z_i + Y^l + n_i^l} = \frac{\bar{z} + Y^h}{\bar{z} + Y^l}.$$

Combining the above with the budget constraint written as $qn_i^h = -n_i^l$, allows one to determine optimal investment into security h

$$n_i^h = \frac{(Y^h - Y^l)(z_i - \bar{z})}{q \cdot (\bar{z} + Y^h) + \bar{z} + Y^l}. \tag{17}$$

Notice that $n_2^h \geqslant 0$ as conjectured because $z_2 > \bar{z}$. By direct differentiation, investment into security h depends on π^l as follows

$$\frac{dn_i^h}{d\pi^l} = -\frac{(Y^h - Y^l)(z_i - \bar{z})(\bar{z} + Y^h)}{[q(\bar{z} + Y^h) + \bar{z} + Y^l]^2} \cdot \frac{dq}{d\pi^l}.$$

By lemma 3, $n_2^h > 0$, $dn_2^h/d\pi^l > 0$ and $n_1^h < 0$, $dn_1^h/d\pi^l < 0$. This means the portfolios grow in size as π^l rises. This is the channel through which π^l affects the equilibrium set.

Figure 4 plots the optimal portfolios assuming CRRA preferences. The portfolios rise with risk-aversion because agents demand more insurance.

C. *Creating and destroying equilibria*

This section will show that there is a number $\bar{\pi}^l \in (0, 1)$ so that equation (8) becomes

$$Y^s = \begin{cases} 0 & \text{if } s = l \\ \alpha + 1 & \text{if } s = h \text{ and } \pi^l \leqslant \bar{\pi}^l \\ 0 & \text{if } s = h \text{ and } \pi^l > \bar{\pi}^l \end{cases}. \tag{18}$$

Then similarly to Lucas's (1977) claim that the distribution of money injections changes the slope of the Phillips curve, here the response of Y to s depends on the distribution of the sunspot s. That is, changing π^l changes how s affects Y.

If equilibrium H is destroyed, equation (18) implies that $Y^s = 0$ with probability 1. There is no financial trade because agents consume only once and there is only one economic outcome. If agents assumed that $s = h$ would lead to everyone setting $x = 1$, and hence that outcome H would occur with a positive probability,

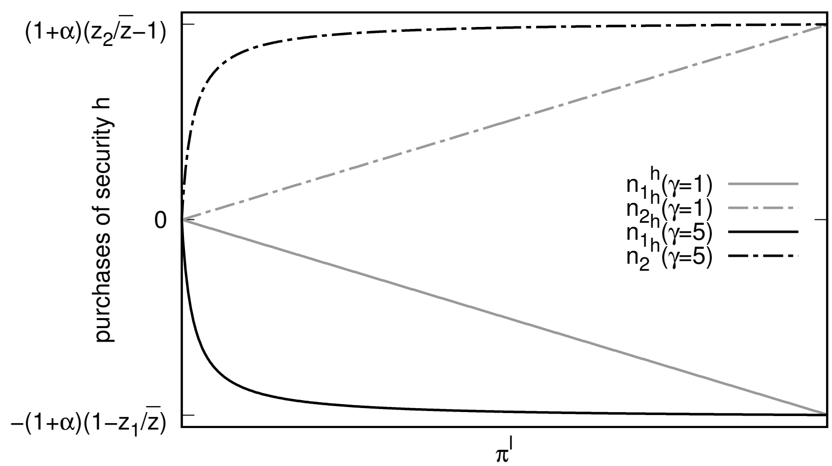


Figure 4. : Optimal portfolios for different levels of risk-aversion under CRRA preferences: $U(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$.
Parameters are reported in Table A1 but $f_1 = f_2 = 0.5$

then they would take up financial positions that would eliminate work incentives for the rich and invalidate that H outcome.

Under financial autarky, L and H equilibria exist under the following four conditions: [5]

$$U(z_1 + \alpha) - \kappa \leqslant U(z_1), \tag{19a}$$

$$U(z_2 + \alpha) - \kappa \leqslant U(z_2). \tag{19b}$$

$$U(z_1 + 1 + \alpha) - \kappa \geqslant U(z_1), \tag{20a}$$

$$U(z_2 + 1 + \alpha) - \kappa \geqslant U(z_2). \tag{20b}$$

As financial markets open, the set of equilibria may change. Financial payments to the endowment-rich in equilibrium H destroy their incentives to work, thus, destroying the equilibrium. The need of the endowment-rich to make financial payments in equilibrium L may prompt them to work.

Our argument below reveals that the endowment-rich is the pivotal agent in the sense that it is her incentives that could change the equilibrium set. This is intuitive because the endowment-rich accept consumption that is more volatile consumption than their income. Consumption of the endowment-rich in equilibrium L falls relative to financial autarky, and this raises their incentives to work. Their consumption in equilibrium H rises, and their incentives to work weaken.

In contrast, consumption volatility of the endowment-poor falls, and their sup-

[5] We report all four inequalities for completeness, but two are redundant.

port for both equilibria strengthens.

Under financial autarky, the conditions for existence of equilibria H and L do not depend on (π^l, π^h). However, with financial markets open this is no longer true.

Destroying equilibria

We focus on the set of parameters for which both equilibria L and H exist and ask if equilibria can be destroyed after opening financial markets. Proposition 5 gives the answer.

PROPOSITION 5: *If both equilibria exist under financial autarky, opening financial markets can destroy equilibrium H but not equilibrium L.*

PROOF:
Suppose that both the H and L equilibria exist under financial autarky, *i.e.*, the conditions (21) and (22) hold. Because $n_2^l \leqslant 0$, effort incentives of the endowment-poor decrease, *i.e.*, they cannot destroy the L equilibrium. The opposite is true of the endowment-rich, as $n_2^l \leqslant 0$. However, equation (15a) shows that $n_2^l \geqslant -\Delta_z$ and we get

$$U(z_2 + \alpha + n_2^l) - \kappa - U(z_2 + n_2^l) \leqslant U(z_1 + \alpha) - \kappa - U(z_1) \leqslant 0.$$

That is, the endowment-rich still prefer not to work.

Because $n_1^h \leqslant 0$, work incentives of the endowment-poor increase, *i.e.*, they cannot destroy the H equilibrium. The opposite is true of the endowment-rich, as $n_2^h \geqslant 0$. It is possible that the endowment-rich do not support the H equilibrium

$$U(z_2 + \alpha + 1 + n_2^h) - \kappa - U(z_2 + n_2^h) < 0,$$

In particular, when $z_1 < z_2 = z_{max}$, we have that $z_2 + n_2^h > z_{max}$, and

$$U(z_{max} + \alpha + 1 + n_2^h) - \kappa - U(z_{max} + n_2^h) < U(z_{max} + \alpha + 1) - \kappa - U(z_{max}) = 0.$$

Figure 5 shows the impact on the equilibrium set. Region L (H) denotes the set of endowments for which only the L (H) equilibrium exists. The non-shaded top left corner is where neither of the two equilibria exists. In this region there exist equilibria with $\bar{x} \in (0, 1)$. In such equilibria a fraction of individuals of type 1 works while the rest do not.

Restriction on π^l

When financial markets are open, the equilibrium set depends on π^l. The Appendix proves

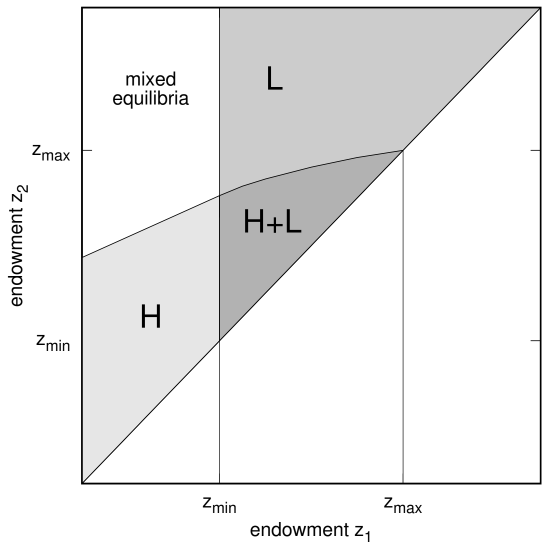


Figure 5. : Equilibrium map when financial markets are open

PROPOSITION 6: *Assume that equilibria H and L both exist under financial autarky. When financial markets open, the two equilibria survive if and only if* $\pi^l \leqslant \bar{\pi}^l$ *for some* $\bar{\pi}^l \geqslant 0$. *Moreover, the set of parameters for which* $\bar{\pi}^l < 1$ *is non-empty.*

The proof that $\bar{\pi}^l < 1$ is possible reveals there must be some endowment inequality in the economy for our argument to work. As inequality rises, as observed around the world, the high outcome is more likely to be destroyed, and its existence can be only supported by more optimism, *i.e.*, by lower $\bar{\pi}^l$.

The above result can be illustrated using figure 5. The area where equilibrium H is destroyed is around the z_2's upper bound. The equilibrium set and π^l are impacted most when inequality is high, *i.e.*, $z_2 - z_1$ is maximal. When there is no inequality, *i.e.*, $z_2 = z_1$, the equilibrium set is unaffected and π^l is unrestricted.

II. Logarithmic Preferences

Under logarithmic preferences, $z_{min} = \frac{\alpha}{e^\kappa - 1}$ and $z_{max} = \frac{\alpha+1}{e^\kappa - 1}$. The set of parameters for which both equilibria exist, denoted with "H+L" in Figure 5, is

$$\mathcal{P} = \left\{ (z_1, z_2, \alpha, \kappa) : \frac{\alpha}{e^\kappa - 1} \leqslant z_1 \leqslant z_2 \leqslant \frac{\alpha + 1}{e^\kappa - 1} \right\}. \tag{21}$$

Next, we derive the optimal portfolios and upper bound on π^l. Having an analytical expression allows us providing further insights into how it depends on the economic environment.

A. Optimal portfolios

In this section we derive optimal portfolios under logarithmic preferences. Then, we analyze the equilibrium set and the supporting beliefs π^l.

We start with optimality condition (11), which simplifies to

$$\frac{z_1 + n_1^l}{z_1 + \alpha + 1 + n_1^h} = \frac{z_2 + n_2^l}{z_2 + \alpha + 1 + n_2^h} = \frac{\pi^l}{\pi^h} q,$$

and implies

$$q = \frac{\pi^h}{\pi^l} \frac{\bar{z}}{\bar{z} + \alpha + 1}.$$

Using the budget constraints and the market clearing conditions lets us solve for the optimal portfolios[6]

$$n_2^l = -\pi^h f_1 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1}, \quad n_1^l = \pi^h f_2 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1}, \tag{22a}$$

$$n_2^h = \pi^l f_1 \Delta_z \frac{\alpha + 1}{\bar{z}}, \quad n_1^h = -\pi^l f_2 \Delta_z \frac{\alpha + 1}{\bar{z}}, \tag{22b}$$

where $\Delta_z \equiv z_2 - z_1$. The payoff to the rich n_2^h is positive as conjectured.

B. Restricting equilibrium values of π^l

The binding inequality is the incentive of the rich to exert effort that holds when $\frac{\alpha+1}{e^\kappa - 1} \geqslant z_2 + n_2^h$. Let

$$\delta = \frac{1}{e^\kappa - 1} \tag{23}$$

which is roughly equal to $1/\kappa$ when the latter is small. After substituting the formula for n_2^h we obtain

$$\pi^l \leqslant \frac{(\alpha + 1)\delta - z_2}{(\alpha + 1) f_1 \Delta_z / \bar{z}} \equiv \bar{\pi}^l. \tag{24}$$

Together with the condition for the existence of the two equilibria under financial autarky, inequality (24) is the restriction on equilibrium beliefs under which both equilibria exist when financial markets are open. Intuitively, π^l cannot be too high as then the endowment-rich type 2 individuals would not work in equilibrium H and the latter ceases to exist. This happens because as π^l grows the

[6] We use market clearing conditions to determine optimal purchases of securities by type-1 individuals: $n_1^s = -(f_2/f_1) n_2^s, s \in \{l, h\}$.

relative prices q and n_2^h rise. But when a payoff in any state rises, effort incentives weaken. The restriction on π^l could also be vacuous, *e.g.*, when $\Delta_z = 0$, or it could be prohibitive, *e.g.*, when $z_2 = (\alpha + 1)\delta$.

The term $(\alpha + 1)\delta - z_2$ is the largest trade that does not destroy type-2's incentives to work. The term $(\alpha + 1)\Delta_z/\bar{z}$ determines the size of the payoff to endowment-rich in equilibrium H, see equation (22b).[7] If there were no heterogeneity, $\Delta_z/\bar{z}$ would be zero and there would be no trade; so, any π^l would do. The term $(\alpha+1)f_1$ is the additional income earned by the poor when equilibrium H is selected. The larger it is the stronger are trading motives and, hence, the higher the chances of destroying equilibrium H.

Figure 6 illustrates the relation between $\bar{\pi}^l$ and (α, δ). Notice that as α and/or δ rise equilibrium L disappears. Similarly, when α and/or δ fall, equilibrium H disappears. For intermediate values of (α, δ) the figure plots the limit on the probability of equilibrium L. When α and/or δ are high, but not enough to destroy equilibrium L, the probability of equilibrium L is unrestricted. In this case the high-endowment type-2 individuals have a substantial " insurance capacity" and provide for the low-endowment individuals while continuing to work. This area corresponds to the plateau in figure 6.a.

Figure 6.b plots contours of $\bar{\pi}^l$. To have multiple equilibria with trading of assets we need $\delta \geqslant z_2/(1 + \alpha)$. When the latter holds at equality, $\bar{\pi}^l = 0$. The upper bound $\bar{\pi}^l$ is linear in δ and hyperbolic in α. It rises with δ that is inversely related to the cost of working, κ.

– As α rises, two effects are operational. First, it is more difficult to destroy equilibrium H because output rises. Second, trades rise as they are proportional to $(1 + \alpha)$, which is the rise in aggregate consumption between the L and H equilibria. However, financial payoffs of any individual cannot exceed $(1 + \alpha)$, and the first effect dominates.

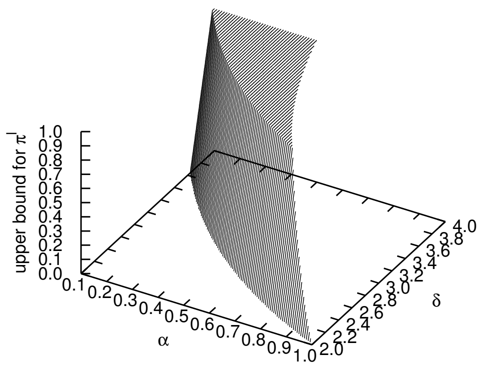
– As κ rises, equilibrium H is more difficult to support and a smaller range of probabilities are consistent with an equilibrium. In the region where $\pi^l < \bar{\pi}^l$, this parameter has no effect on the size of financial trades or equilibrium prices.

Financial markets do not create new equilibria

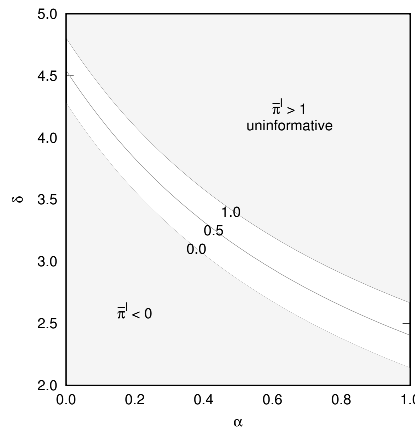
For completeness, Appendix A3 shows that financial markets do not create new equilibria. Formally, Appendix A3 proves the following result:

PROPOSITION 7: *Opening financial markets cannot create equilibrium H (L) equilibrium if only equilibrium L (H) existed under financial autarky.*

[7] The fraction f_1 in the equation (24) suggests that the incentives of the endowment poor matter, but that is not the case because $z_2 - \bar{z} = f_1\Delta_z$.



(a) Relationship between $\bar{\pi}^l$ and (α, δ).



(b) Contours of $\bar{\pi}^l$

Figure 6. : Upper bound on probability of the disaster state π^l.
Parameters: logarithmic utility, $f_1 = f_2 = 0.5, z_1 = 2, z_2 = 4$.

C. Welfare

The effect of financial markets on welfare is ambiguous. For $\pi^l \leqslant \bar{\pi}^l$, welfare rises because agents insure each other. For $\pi^l > \bar{\pi}^l$ welfare declines. Figure 7 provides an illustration. It plots the welfare of the rich and poor agents in the economy with and without financial trade. Welfare is decreasing in π^l, as shown in appendix A2.

If financial markets are closed both the high and the low outcomes are possible, and the welfare of the two types is shown by the dotted lines

$$W_i = \pi^l U(z_i) + \left(1 - \pi^l\right) [U(z_i + 1 - \alpha) - \kappa] . \tag{25}$$

As π^l rises towards $\bar{\pi}^l$, welfare declines, but opening financial markets improves everyone's welfare; the solid line for W_i is above the dotted line for each i. The opposite is true for $\pi^l \geqslant \bar{\pi}^l$.

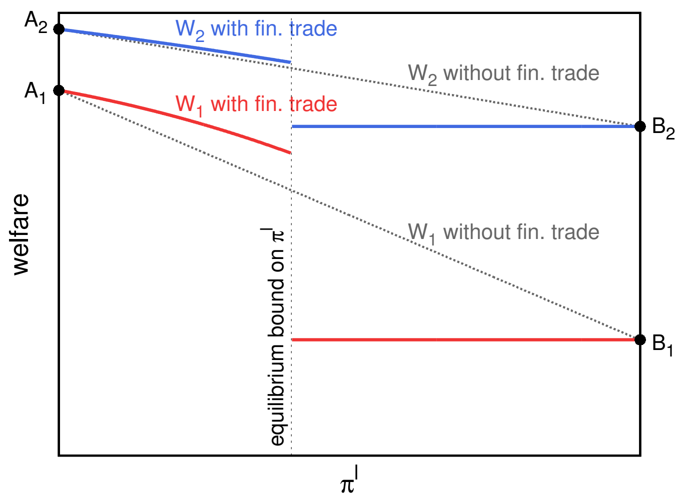


Figure 7. : Welfare with (solid lines) and without (dotted lines) financial markets.

Our result differs from Hart (1975) who showed that partial completion of financial markets can have a negative welfare effect. Hart had an exchange economy with no external effects in consumption, whereas we have a production economy with external effects, and a full set of state-contingent securities is traded, *i.e.*, financial markets are complete.

D. Distributional effects

Endowment inequality can be measured by $\Delta_z/\bar{z}$. Rising inequality reduces $\bar{\pi}^l$. The more dispersed endowments are, the larger are incentives to trade in equilibrium for then the rich value consumption much less than the poor. On the other hand, when endowments are similar there is little incentives to trade. In this case the set of possible sunspot equilibria is unaffected as $\bar{\pi}^l \geqslant 1$ is not restrictive. Thus, when dispersion is small, $\Delta_z/\bar{z} \leqslant \delta - z_2/(\alpha+1)/f_1$, opening financial markets has no effect on the probability of equilibrium L.

Group sizes also matter. If $f_1 > f_2$, each individual from a larger poor type saved one unit then individuals in the other, smaller, group would receive more than one unit. For this reason, the payment to the rich in outcome h rises with f_1, and it may destroy equilibrium H.

In societies with a small fraction of poor individuals, opening financial markets is unlikely to affect the equilibrium set. A significant improvement in risk-sharing across equilibria can be achieved, as it costs little for the populous high-endowment group to insure a small group of poor. Formally, $|c_1^H - c_1^L|$ falls as f_2 rises. Low-endowment individuals demand insurance, and high-endowment individuals are willing to provide it, regardless of the group proportions (f_1, f_2).

E. Risk-aversion

Log preferences greatly simplify our derivations, but our results carry over to the case with any CRRA utility function.[8] What role does risk-aversion play? Figure 4 plots purchases of Arrow security h of the poor type-1 and the rich type-2 individuals as functions of π^l. As the risk-aversion coefficient rises from $\gamma = 1$ to $\gamma = 5$ the position in the Arrow security H of the rich type-2 rises faster. More risk-averse individuals opt for a more equitable allocation that is supported by taking larger portfolio positions. Such a large transfer, however, violates the rich type's "incentives constraint" – the rich type stops working and only equilibrium L survives. Thus, as the level of risk-aversion rises $\bar{\pi}^l$ falls. As shown in figure 1, the set of parameters for which equilibrium H exists shrinks.

III. Infinite-horizon version

Time is infinite and indexed by $t = 0, 1, 2, ...$ We assume that sunspot s is an i.i.d. random variable that takes on two values $s \in \{l, h\}$ with probabilities π^l and π^h. There are two types of agent indexed by $i \in \{1, 2\}$. A type i agent receives a constant endowment, z_i in each period.

A. No financial markets

With no financial markets, only the sunspot realization s matters for decisions. An agent has utility of consumption $U(c)$, and takes decision rules of others as summarized by $\bar{x}^s$ as given. A type-i agent's lifetime utility in state s, A_i^s, satisfies

$$A_i^s = \max_x \left\{ U(z_i + (\alpha + \bar{x}^s)x) - \kappa x + \beta \sum_{j \in \{l,h\}} \pi^j A_i^j \right\}. \tag{26}$$

The expected continuation value $\sum_{j \in \{l,h\}} \pi^j A_i^j$ does not depend on agent's actions. As a consequence, the conditions for equilibrium L and H existence are the same as in the one-period model.

[8] We do not provide details but we point out that the ratio of individual consumption and the aggregate supply of goods across the two sub-game equilibria are all the same.

B. Financial markets

Each period financial markets trade two securities. Assuming that current state is s, security j trades at Q^{sj} and pays one unit of consumption if state $s' = j$ realizes tomorrow. Each agent starts with a zero financial position in period $n_{i,0} = 0$. Trading in financial markets starts in period 0, production, consumption, and endowments flow start at $t = 1$.

Decisions.—Agents take as given asset prices (Q^{sl}, Q^{sh}) and the decision rules of others, as summarized by $\bar{x}^s$. The budget constraint of a type i agent who starts with financial position n in state s and chooses effort x is

$$z_i + (\alpha + \bar{x}^s)x + n = c_i^s + \sum_{j \in \{l,h\}} Q^{sj} n'^j. \tag{27}$$

Solving for consumption from (27), optimal lifetime utility is

$$V_i^s(n) = \max_{x,n'} \Bigg\{ U((\alpha + \bar{x}^s)x + z_i + n - \sum_{j \in \{l,h\}} Q^{sj} n'^j) - \kappa x + \beta \sum_{j \in \{l,h\}} \pi^j V_i^j(n'^j) \Bigg\}, \tag{28}$$

Since markets are dynamically complete and $U(c) = c^{1-\gamma}/(1-\gamma)$ is homothetic, each agent consumes a constant fraction of aggregate income. As a result, price of any security j depends on s but not on time. Let

$$D = \left(\frac{\bar{z} + \alpha + 1}{\bar{z}} \right)^{-\gamma}. \tag{29}$$

LEMMA 8: *A type i agent consumes a constant share ϕ^i of aggregate income so that*

$$c_i^s(n) = \phi_i(\bar{z} + y^s), \tag{30}$$

where

$$\phi_i = \frac{\pi^l z_i + \pi^h (z_i + y^h) D}{\pi^l \bar{z} + \pi^h (\bar{z} + y^h) D} = \begin{cases} < 1, & i = 1 \\ > 1, & i = 2 \end{cases}. \tag{31}$$

Security prices are

$$Q^{sj} = \beta \pi^j \left(\frac{\bar{z} + y^j}{\bar{z} + y^s} \right)^{-\gamma}. \tag{32}$$

The Appendix also shows that the rich agent receives a net financial transfer from the poor agent when $s = h$, as in the one-period model.

Financial equilibrium H.—In any equilibrium, optimal consumption does not depend on κ. This allows one to find κ such that equilibrium H exists under financial autarky but not when the financial markets are open, as stated in the next proposition:

PROPOSITION 9 (Equilibrium H can be destroyed): *For any $z_1 < z_2$, any $\alpha > 0$ and any $\pi^l \in (0, 1)$, there exists $\kappa > 0$ such that equilibrium H exists if financial markets are closed, but not if they are open.*

Further, the following corollary states that equilibrium H destruction is more likely when π^l is high, as in the one-period model.

COROLLARY 10: *Suppose that for some (z_1, z_2, y^h, κ) and $\pi^l \equiv \hat{\pi} < 1$ equilibrium H is destroyed after opening financial markets. Then, there exists a value $\bar{\pi}^l_\infty \leqslant \hat{\pi}$ such that equilibrium H is destroyed for all $\pi^l \geqslant \bar{\pi}^l_\infty$ but it continues to exist if $\pi^l < \bar{\pi}^l_\infty$.*

Log utility case.—We cannot establish that $\bar{\pi}^l_\infty < 1$ in general. The upper bound on π^l solves the following equation

$$U(z_2 + 1 + \alpha + b^h_2) - U(z_2 + b^h_2) = \kappa. \tag{33}$$

where b^h_2 is the net consumption of wealth in state h that depends on π^l and that is given by equation (A17). With $U(c) = \log(c)$, we have $b^h_2 = \pi^l(z_2 - \bar{z})(1 + \alpha)/\bar{z}$, and the static bound $\bar{\pi}^l$ solves the above equation. This fact implies that equilibrium H is destroyed at the $\pi^l = \bar{\pi}^l$, and by corollary 10, the upper bound on π^l in the infinite case, $\bar{\pi}^l_\infty$, is smaller or equal to that in the static case

$$\bar{\pi}^l_\infty \leqslant \bar{\pi}^l.$$

See figure 1 for the comparison of the numerically computed upper bounds on π^l for the static and infinite horizon models.

IV. A real-shock version

We now show that our results extend to models with unique equilibria where sunspots play no role.

A. *Unique equilibrium with real shocks*

Suppose that there is no external effect and assume that the production function is

$$\text{agent } i\text{'s output} = \alpha x_i$$

with $x_i \in \{0,1\}$ and with the effort cost still κx_i and a two-point wealth distribution at (z_1, z_2). Additionally, suppose that α is a random variable assuming two values $\alpha_2 \gg \alpha_1 > 0$ and the distribution as follows

$$\alpha = \begin{cases} \alpha_1 & \text{with prob. } \mu \\ \alpha_2 & \text{with prob. } 1-\mu \end{cases} . \tag{34}$$

Equilibrium can then be unique for each α realization and sunspots do not play a role.

Boom.—Let α_2 be such that everyone wants to work, which is true when

$$U(z_2 + \alpha_2) - U(z_2) \geqslant \kappa. \tag{35}$$

The poor then also want to work, and this is the economic boom "outcome H" with aggregate output $Y^{\alpha_2} = \alpha_2$.

Disaster.—If α_1 is low enough, no one wants to work. This is true when

$$U(z_1 + \alpha_1) - U(z_1) < \kappa. \tag{36}$$

The rich then also do not want to work, and this is the economic disaster "outcome L" with aggregate output $Y^{\alpha_1} = 0$. Disaster size is $Y^{\alpha_2} - Y^{\alpha_1} = \alpha_2$.

We assume that (35) and (36) both hold so that equilibrium is unique conditional on α.

Financial markets.—The financial markets trade two Arrow securities that pay based on the realization of α. The poor still desire to insure against state α_1, and they achieve it by selling Arrow security α_2 that pays in state α_2. With logarithmic utility, the optimal portfolios are

$$n_j^{\alpha_2} = \mu(z_j - \bar{z})\alpha_2/\bar{z}, \tag{37a}$$

$$n_j^{\alpha_1} = -(1-\mu)(z_j - \bar{z})\alpha_2/(\bar{z} + \alpha_2). \tag{37b}$$

Equations (37a) and (37b) show that the higher is the "disaster probability" μ, the larger the payment $n_2^{\alpha_2}$ the rich receive in state α_2. Suppose that if the poor were to transfer their entire endowment z_1 to the rich, the rich would not want to work in the high state so that

$$U(z_1 + z_2 + \alpha_2) - U(z_2) < \kappa.$$

Then there must exist $\bar{\mu} < 1$ such that the post-transfer inequality (35) would not hold for $\mu \geqslant \bar{\mu}$. In the latter case, only poor work and aggregate output is

$Y = f_1\alpha_2$. This is a middle "outcome M." In turn, optimal portfolios change to

$$n_2^{\alpha_2} = \mu z_2 f_1 \alpha_2/\bar{z}, \tag{38a}$$

$$n_2^{\alpha_1} = -(1-\mu) z_2 f_1 \alpha_2/(\bar{z} + f_1\alpha_2). \tag{38b}$$

There is a discrete rise in payoff to the rich that reinforces their incentives not to work.

$$n_2^{\alpha_2}(Y = \alpha_2) < n_2^{\alpha_2}(Y = f_1\alpha_2).$$

Figure 8 plots the optimal portfolio of the rich and shows the pairs of outcomes can occur as a function of μ. The real-shock version of the model places an upper bound on the size of the disaster as well as its probability. In contrast to the sunspot model, when $\mu > \bar{\mu}$, the size of the disaster does not fall to zero but to $f_1\alpha_2$. There is still a financial equilibrium but with a lower positive output, while there would be no positive-output equilibrium in the model described in section I.

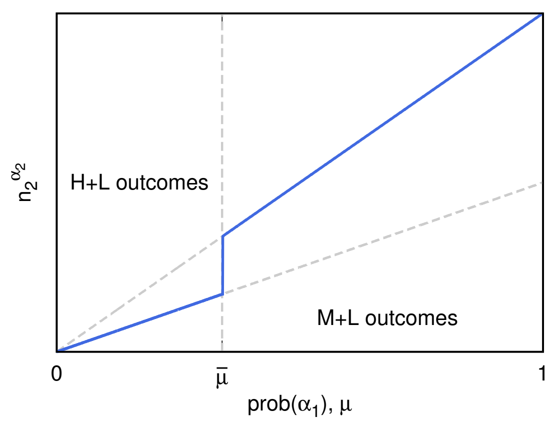


Figure 8. : Rich agent's holding of α_2-security and the possible outcomes as a function of the probability of low-productivity state α_1

B. Sunspots and real shocks together

If we add the real shock in (34) but revert to the original production function $(\alpha + \bar{x})x_i$, then the externality may create more than one equilibrium in each α state. We may assume that (α_1, α_2) are both in the range where equilibria L and the H both exist. The disaster state then is $(s, \alpha) = (l, \alpha_1)$.

Financial markets trade four Arrow securities, each paying in one of the four realizations of (s, α). The poor want to insure against the disaster state (l, α_1), which probability is $\pi^l \mu$. If this probability is high enough, transfers from the poor to the rich may again be large enough to eliminate the high equilibria or reduce output when $\alpha = \alpha_2$.

V. Conclusion

In a model in which multiple Pareto-ranked equilibria may arise, we showed that when agents wish to hedge against the risk of a disaster, opening financial markets can make things worse. Rather than allowing agents to insure themselves, financial markets can destroy the H equilibrium and make the disaster inevitable, with no insurance provided. The welfare effects of finance are therefore ambiguous; if the equilibrium set stays intact welfare rises but if the high equilibrium is destroyed, welfare falls.

We showed that the probability of disasters depends negatively on the degree of risk aversion and that inequality in wealth and consumption is pro-cyclical which evidence supports. These implications continue to hold in an infinitely repeated version of the model, as well as in an extended version of the model that has real shocks, a unique equilibrium, and in which asset trading is on real shocks and not on sunspots.

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Mathematical Appendix

A1. Proof of proposition 6

PROOF:

According to proposition 5, only equilibrium H can be destroyed and, thus, we need to analyze its existence. Since it exists under financial autarky, we have

$$U(z_i + 1 + \alpha) - \kappa - U(z_i) \geqslant 0. \tag{A1}$$

At $\pi^l = 0$ the payoff $n_1^h = n_2^h = 0$, and equilibrium H continues to exist. At

$\pi^l > 0$, the gain from exerting effort in equilibrium H is

$$\Delta_i \equiv U(z_i + 1 + \alpha + n_i^h) - \kappa - U(z_i + n_i^h). \tag{A2}$$

Since $n_1^h \geqslant 0$, $\Delta_1 \geqslant 0$. Hence, we only need to analyze the incentives of the endowment-rich individuals.

The derivative of the gain Δ_2 is

$$\frac{d\Delta_2}{d\pi^l} = \left[U'(z_2 + 1 + \alpha + n_2^h) - U'(z_2 + n_2^h)\right]\frac{dn_2^h}{d\pi^l} < 0.$$

It is negative because the term in the square brackets is negative and the derivative of n_2^h is positive, *i.e.*, the payoff to the endowment-rich individual is decreasing with π^l as explained in section I.B. The incentives for the wealthy to exert effort decrease, as the probability of equilibrium L increases. This means that there exists a upper bound on π^l, proving the first statement of this proposition.

The upper bound is informative if the rich do not have incentives to work at $\pi^l = 1$. Using (16), consumption of the rich in state h, when it occurs with probability 0, is $\tilde{c}_2 = z_2/\bar{z}\cdot(\bar{z}+1+\alpha)$. If the rich stop working, their consumption declines by $1+\alpha$. Hence, work incentives of the rich are destroyed when

$$U\left(\frac{z_2}{\bar{z}}(\bar{z}+1+\alpha)\right) - \kappa < U\left(\frac{z_2}{\bar{z}}(\bar{z}+1+\alpha) - 1 - \alpha\right).$$

The above inequality can be rewritten to resemble the condition for the rich to exert effort in financial autarky (7), which is this proposition's assumption,

$$U\left(z_2 + \frac{z_2}{\bar{z}}(1+\alpha)\right) - \kappa < U\left(z_2 + \left(\frac{z_2}{\bar{z}} - 1\right)\cdot(1+\alpha)\right). \tag{A3}$$

If z_2 equalled $\bar{z}$, the above inequality would contradict (7). However, it is possible that both (A3) and (7) hold when $z_2 > \bar{z}$. In particular, take $z_2 = z_{max}$ such that (7) holds at equality. Then, (A3) holds strictly because $U\left(z_2 + \frac{z_2}{\bar{z}}(1+\alpha)\right) - U\left(z_2 + \left(\frac{z_2}{\bar{z}} - 1\right)(1+\alpha)\right)$ is decreasing in z_2. Because U is continuous, the two inequalities hold strictly for z_2 near z_{max}. This proves that the set of parameters for which $\bar{\pi}^l < 1$ is non-empty.

A2. Proof that welfare decreases in π^l

LEMMA 11: *Assume that $U = \log$. If both the L and H equilibria exist, welfare level W_i decreases in π^l for $i = 1, 2$.*

PROOF:

The following is true for any utility function:

$$\frac{dW_1}{d\pi^L} = \underbrace{U(z_1+n_1^L) - U(z_1+\alpha+1+n_1^H)}_{negative} + \underbrace{\pi^L u'(z_1+n_1^L)\frac{dn_1^L}{d\pi^L} + (1-\pi^L)U'(z_1+\alpha+1+n_1^H)\frac{dn_1^H}{d\pi^L}}_{both\ terms\ are\ negative} < 0.$$

Letting $U(c) = \log(c)$ one obtains:

$$\begin{aligned}\frac{dW_2}{d\pi^L} =& \underbrace{U(z_2+n_2^L) - U(z_2+\alpha+1+n_2^H)}_{negative} \\ &+ \underbrace{\pi^L u'(z_2+n_2^L)\frac{dn_2^L}{d\pi^L} + (1-\pi^L)U'(z_2+\alpha+1+n_2^H)\frac{dn_2^H}{d\pi^L}}_{both\ terms\ are\ positive} \\ =& U(z_2+n_2^L) - U(z_2+\alpha+1+n_2^H) \\ &+ \pi^L\pi^H f_1\Delta_z(\alpha+1)\left[\frac{U'(z_2+n_2^L)}{\bar{z}+\alpha+1} + \frac{U'(z_2+\alpha+1+n_2^H)}{\bar{z}}\right],\end{aligned}$$

where the last equality relies on the optimal portfolios derived in (22b) and (22a). Then by the concavity of U and the fact that $U'(z_2+\alpha+1+n_2^H)/U'(z_2+n_2^L) = (\bar{z}+\alpha+1)/\bar{z}$ we get

$$\begin{aligned}\frac{dW_2}{d\pi^L} \leqslant& -U'(z_2+a+1+n_2^H)(\alpha+1) \\ &+ \pi^L\pi^H f_1\Delta_z(\alpha+1)\left[\frac{U'(z_2+n_2^L)}{\bar{z}+\alpha+1} + \frac{U'(z_2+\alpha+1+n_2^H)}{\bar{z}}\right] \\ =& -U'(z_2+a+1+n_2^H)(\alpha+1) + 2\pi^L\pi^H f_1\Delta_z(\alpha+1)U'(z_2+\alpha+1+n_2^H)/\bar{z} \\ =& U'(z_2+a+1+n_2^H)(\alpha+1)[-1+2\pi^L\pi^H f_1\Delta_z/\bar{z}] < 0.\end{aligned}$$

A3. Proof that equilibria cannot be created

Suppose that without financial markets only equilibrium L or equilibrium H exists. Could the other equilibrium be "created" by opening financial markets? The following proposition states that it is not the case.

PROPOSITION 12: *If only the H (L) equilibrium exists under financial autarky, opening financial markets cannot create the L (H) equilibrium.*

PROOF:

Suppose only the L equilibrium exists under financial autarky. In this case, one or both of the inequalities holds so that at least one type is not willing to work

$$U(z_1+\alpha+1)-\kappa<U(z_1),$$
$$U(z_2+\alpha+1)-\kappa<U(z_2).$$

When financial markets open, the incentives of the endowment-rich to work decrease because $n_2^h \geqslant 0$. Hence, the H equilibrium continues to be non-viable. Mathematically, the concavity of U implies that the gain from working is negative:

$$U(z_2+\alpha+1+n_2^h)-\kappa-U(z_2+n_2^h)\leqslant U(z_2+\alpha+1)-\kappa-U(z_2)<0.$$

Suppose only equilibrium H exists under financial autarky. In this case, at least one of the inequalities below holds

$$U(z_1+\alpha)-\kappa>U(z_1),$$
$$U(z_2+\alpha)-\kappa>U(z_2).$$

When financial markets open, the incentives of the endowment-rich to work increase because $n_2^l \leqslant 0$. Hence, the L equilibrium continues to be non-viable. Mathematically, the concavity of U implies that the gain from working is positive

$$U(z_2+\alpha+n_2^l)-\kappa-U(z_2+n_2^l)\geqslant U(z_2+\alpha)-\kappa-U(z_2)>0.$$

A4. Proofs for the infinite horizon model

Proof of Lemma 8

We start with optimization problem of agent i as stated in equation (28). The envelope theorem implies that in (28), $V'(n)=U'(c)$, and so the first-order condition w.r.t. $n_j^{j\prime}$ yields the consumption Euler equation

$$Q^{sj}=\beta\pi^j\frac{U'\left(c_i^{\prime j}\right)}{U'\left(c_i^s\right)}. \tag{A4}$$

Because each agent faces the same asset prices, the marginal utilities of the two types must grow at the same rate

$$\frac{U'\left(c_1^{\prime j}\right)}{U'\left(c_1^s\right)}=\frac{U'\left(c_2^{\prime j}\right)}{U'\left(c_2^s\right)},\quad \forall\left(s,j\right).$$

Homotheticity of U then implies that the consumption growth of both types must also be the same, and must equal the growth of the aggregate supply of goods in all state pairs:

$$\frac{c_1'^{\,j}}{c_1} = \frac{c_2'^{\,j}}{c_2} = \frac{\bar{z} + y^j}{\bar{z} + y^s}, \quad \forall (s, j). \tag{A5}$$

Individual consumption must then be proportional to the aggregate supply of goods:

$$c_i = \phi_i(\bar{z} + y^s), \quad i \in \{1, 2\}\,. \tag{A6}$$

Eqs. A4 and (A6) imply (32).

Expected marginal utilities at $t = 1$ (when z is received for the first time) are equated to costs Q^{0j} and the envelope condition then yields

$$\frac{\pi^h U'(c_i^h)}{\pi^l U'(c_i^l)} = \frac{\pi^h}{\pi^l}\left(\frac{\bar{z} + y^h}{\bar{z} + y^l}\right)^{-\gamma} = \frac{Q^{0h}}{Q^{0l}}.$$

Because there is no consumption in the opening period, one price at $t = 0$ needs to be normalized. We thus set

$$Q^{0l} = \pi^l, \quad \text{and} \quad Q^{0h} = \pi^h D, \tag{A7}$$

so that

$$D = \frac{Q^{0h}}{\pi^h} = \left(\frac{\bar{z} + \alpha + 1}{\bar{z}}\right)^{-\gamma}. \tag{A8}$$

Then for $t = 0$, security prices are:

$$Q^{0l} = \pi^l, \tag{A9a}$$

$$Q^{0h} = \pi^h D. \tag{A9b}$$

For $t \geqslant 1$, security prices are state-dependent but not time-dependent. The present discounted value of aggregate income, that depends on state s, solves the following system of equations

$$I^0 = \pi^l I^l + \pi^h D I^h, \tag{A10a}$$

$$I^l = \bar{z} + \beta\pi^l I^l + \beta\pi^h D I^h, \tag{A10b}$$

$$I^h = \bar{z} + y^h + \beta\pi^l I^l / D + \beta\pi^h I^h. \tag{A10c}$$

The solution is

$$I^0 = [\pi^l \bar{z} + \pi^h(\bar{z} + y^h)D]/(1-\beta), \tag{A11a}$$

$$I^l = [(1-\beta\pi^h)\bar{z} + \beta\pi^h(\bar{z} + y^h)D]/(1-\beta), \tag{A11b}$$

$$I^h = [\beta\pi^l \bar{z}/D + (1-\beta\pi^l)(\bar{z} + y^h)]/(1-\beta). \tag{A11c}$$

Derived in the same way, present discounted value of individual income is

$$I_i^0 = [\pi^l z_i + \pi^h(z_i + y^h)D]/(1-\beta), \tag{A12a}$$

$$I_i^l = [(1-\beta\pi^h)z_i + \beta\pi^h(z_i + y^h)D]/(1-\beta), \tag{A12b}$$

$$I_i^h = [\beta\pi^l z_i/D + (1-\beta\pi^l)(z_i + y^h)]/(1-\beta). \tag{A12c}$$

The consumption share of an agent i is

$$\phi_i = I_i^0/I^0 = \frac{\pi^l z_i + \pi^h(z_i + y^h)D}{\pi^l \bar{z} + \pi^h(\bar{z} + y^h)D}. \tag{A13}$$

Choice of x.—Equilibrium H exists if for $i \in \{1,2\}$

$$\Delta V_i^h \equiv \max_{n'} \left\{ U(\alpha + 1 + z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) - \kappa + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\}$$
$$- \max_{n'} \left\{ U(z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} \geqslant 0, \tag{A14a}$$

and equilibrium L exists if for $i \in \{1,2\}$

$$\Delta V_i^l \equiv \max_{n'} \left\{ U(\alpha + z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) - \kappa + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\}$$
$$- \max_{n'} \left\{ U(z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} \leq 0. \tag{A14b}$$

While the optimal n' will generally differ if the agent deviates from his equilibrium choice of x, we can use a sufficient condition if the equilibrium portfolio choices remain feasible following the deviation. Feasibility can be an issue if the deviation is from $x = 1$ to $x = 0$ which then entails a loss of income. We will show in equation (A17)) that if the rich deviate, they can still hold their pre-deviation portfolio and still have strictly positive consumption. In that case, we have the following upper bound on the return to exerting effort

$$\Delta V_2^h \leqslant U\left(z_2 + y^h + n_2^h - \sum_{j\in\{l,h\}} Q^{hj} n_2'^j\right) - \kappa - U(z_2 + n_2^h - \sum_{j\in\{l,h\}} Q^{hj} n_2'^j)$$
$$= U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) - \kappa. \tag{A15}$$

DERIVATION OF NET PORTFOLIO PAYOFF

By the budget constraint (27), net portfolio payoff must equal net consumption

$$b_i^s \equiv n_i^s - \sum_j Q^{sj} n_i'^j = c_i^s - (z_i + y^s). \tag{A16}$$

Under the assumptions of lemma 8, net consumption of agent i in state $s = h$ is

$$c_i^h - (z_i + y^h) = \frac{\pi^l (z_i - \bar{z}) y^h}{\pi^l \bar{z} + \pi^h (\bar{z} + y^h) D} = \begin{cases} < 0, & i = 1 \\ > 0, & i = 2 \end{cases}. \tag{A17}$$

That is, the endowment-rich type 2 agent consumes more than his or her income in state $s = h$ or, equivalently, receives a net financial transfer from an endowment-poor type 1 agent.

Intuitively, consumption of the richer type-2 agent is more volatile. He or she suffers from consumption volatility less because since $U''' > 0$, period utility is flatter at higher levels of consumption. ∎

PROOF OF PROPOSITION 9

Suppose that $\Delta A_i^h \geqslant 0, \forall i$. We will show that it is possible to chose κ so that $\Delta V_2^h < 0$.

By equation (A15), we have $\Delta V_2^h \leqslant U(z_2+y^h+b_2^h) - \kappa - U(z_2+b_2^h)$, where $b_2^h > 0$ is the optimal consumption of wealth of type-2 agent in state h. Because $b_2^h > 0$ and U is strictly concave, we have

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < U(z_2 + y^h) - U(z_2).$$

Define $\kappa = 0.5[U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) + U(z_2 + y^h) - U(z_2)] > 0$, for which we have

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < \kappa < U(z_2 + y^h) - U(z_2),$$

or, equivlently,

$$U(z_2 + y^h + b_2^h) - \kappa - U(z_2 + b_2^h) < 0 < U(z_2 + y^h) - \kappa - U(z_2). \tag{A18}$$

The left and right inequality imply, respectively, that $\Delta V_2^h < 0$ and $\Delta A_2^h > 0$.

Because type-2 agent prefers not to work when $s = h$, equilibrium H is destroyed after opening financial markets.

Note that, because all expressions are continuous functions of the parameters, there must exist an open set containing the identified parameters and for which the proposition's statement holds. This proves the proposition.

Proof of corollary 10

Since all expressions are continuous functions of the parameters, there must exist an open set containing the identified parameters and for which the proposition's statement holds. As shown above,

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < \kappa < U(z_2 + y^h) - U(z_2).$$

Since b_2^h in Eq. (A16) increases with π^l, $U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h)$ decreases as π^l increases. Thus, there must exist a value $\hat{\pi}$ such that the left inequality holds for all $\pi^l < \hat{\pi}$ but that it reverses for $\pi^l \geqslant \hat{\pi}$.
For log utility $U = \log$, we solve $\log(z_2 + y^h + b_2^h) - \log(z_2 + b_2^h) = \kappa$ for $\hat{\pi}^l$ as follows

$$\begin{aligned}
\log(z_2 + y^h + b_2^h) - \log(z_2 + b_2^h) = \log(1 + y^h/(z_2 + b_2^h)) &= \kappa \\
\log(1 + y^h/(z_2 + b_2^h)) &= \kappa \\
\delta y^h/(z_2 + b_2^h) &= 1 \\
\delta y^h - z_2 &= b_2^h
\end{aligned}$$

where δ is defined in equation (23). In equation (A8) we see that when $\gamma = 1$, $D = \bar{z}/(\bar{z} + y^h)$. Using (A17) and the fact that b_2^h is increasing in π^l we then get the following upper bound for $\bar{\pi}$:

$$\hat{\pi} = \frac{(\bar{z} + y^h)D}{\frac{(z_2 - \bar{z})y^h}{\delta y^h - z_2} - (\bar{z} - (\bar{z} + y^h)D)} = \frac{\bar{z}\left(\delta y^h - z_2\right)}{(z_2 - \bar{z})y^h}.$$

Since $y^h = \alpha + 1$, $\hat{\pi} = \bar{\pi}^l$ as defined in (24).

A5. *Parameters used by figure 1*

κ	0.40	cost of investing
f_1	0.90	the fraction of endowment-poor
(z_1, z_2)	(0.20, 1.00)	endowments
α	0.06	loss of total output in equilibrium L

Table A1—: Parameters used to produce Figure 1

Equilibrium L cannot be destroyed

Suppose that equilibrium L exists under financial autarky, *i.e.*,

$$\kappa > u(z_i + \alpha) - u(z_i), \quad \forall i.$$

This implies that for any $b > 0$ we have

$$u(z_i + b) - u(z_i + \alpha + b) + \kappa > 0, \quad \forall i. \tag{A19}$$

Let E to denote expectation over state s, and let $\hat{m}$ denote the optimal portfolio chosen by an agent choosing to deviate in equilibrium L. Then

$$\begin{aligned} &\max_m \{u(z_i + n - Qm) + \beta E[V^s(m^s)]\} \\ &\quad - \max_m \{u(z_i + \alpha + n - Qm) - \kappa + \beta E[V^s(m^s)]\} \\ &\quad > u(z_i + n - Q \cdot \hat{m}) - u(z_i + \alpha + n - Q \cdot \hat{m}) + \kappa > 0. \end{aligned}$$

The last inequality follows from (A19) if $n - Q \cdot \hat{m} > 0$, which we establish next.

Independently of whether an agent follows the equilibrium actions or deviates, his or her consumption at the same aggregate rate as anyone else's. Thus, the present discounted value of one's consumption is a constant fraction of the present discounted value of the aggregate income, PY. Then,

$$\begin{aligned} n + PY_i &= \phi_i PY \\ n + PY_i + \alpha &= \hat{\phi}_i PY \end{aligned}$$

and a deviating agent consumes a larger fraction of the aggregate income than his or her non-deviating counterpart

$$\hat{\phi}_i = \phi_i + \alpha/PY. \tag{A20}$$

Next

$$n - Q \cdot \hat{m} = \hat{\phi}_i \bar{z} - z_i - \alpha = \phi_i \bar{z} - z_i + \alpha(\bar{z}/PY - 1) \to_{\beta \to 0} \phi_i \bar{z} - z_i. \tag{A21}$$